Analysis of shear wall structures of variable thickness using continuous connection method

Jacek Wdowicki and Elżbieta Wdowicka
Institute of Structural Engineering, Poznań University of Technology
Piotrowo 5, 60-965 Poznań
e-mail: jacek.wdowicki@put.poznan.pl

Abstract
The paper presents the analysis of shear wall structures of variable thickness using a variant of the continuum method. In the continuous approach the horizontal connecting beams, floor slabs and vertical joints are substituted by continuous connections. The differential equation systems for shear wall structure segments of constant cross-section are uncoupled by orthogonal eigenvectors. The boundary conditions for the whole structure yield the system of linear equations for the determination of all constants of integration. The results obtained by means of this method show good agreement with those available in literature.

Keywords: shear wall structures, variable thickness, continuous connection method, tall buildings

1. Introduction

In the construction of multistorey reinforced concrete buildings, shear wall structures are commonly used for resisting lateral loads due to wind and seismic effects. Two methods appear to be particularly suitable for the analysis of this type of structure, namely, the continuum method [7], [17], [20], [19], [1] and the finite strip method [12], [3]. The continuum method has proved itself to be extremely practical in structural analysis and design of tall buildings [10]. It is quite common that a shear wall may have different thickness along the height of a building. The upper portion of the wall is subjected to much lower stress than the portion near the support. Hence, several reductions of the thickness of the wall, as it goes up, is a common design practice [2]. The application of the continuum method to the analysis of coupled shear walls with abrupt changes in the cross-section has been considered in Ref. [16], [4], [5], [15], [14] with the use of the analytical method of solving differential equations. In Ref. [11], [10] the finite difference method has been used. Methods proposed in Ref. [6], [18] are based on a transfer matrix technique. In Ref. [9] the iterative technique, based on a combination of the finite strip method and the continuum method, has been presented. In Ref. [8] a macro-element for the analysis of coupled shear wall systems has been introduced. Its formulation is based on the classical continuum method. The purpose of the paper is to present the effective algorithm of the analysis of shear wall structures of variable thickness using the continuous connection method.

2. Governing differential equations

Equation formulations for a three-dimensional continuous model of the shear wall structure with the constant cross-section have been given in Ref. [20]. A structure, which changes its thickness along the height, can be divided into segments, each one having the constant cross-section. For k-th segment the differential equations can be stated as follows:

\[ z \in (h_{k-1}, h_k) > B_{N,k}(z) - A_{N,k}(z) = f_{N,k}(z), \quad (1) \]

where \( B_{N,k} \) is \( n_n \times n_n \) diagonal matrix, containing continuous connection flexibilities, \( A_{N,k} \) is \( n_n \times n_n \) symmetric, positive definite matrix, dependent on a structure, \( n_n \) is the number of continuous connections which substitute connecting beam bands and vertical joints, \( N_{N,k}(z) \) is a vector containing unknown functions of the shear force intensity in continuous connections and \( f_{N,k}(z) \) is a vector formed on the basis of given loads for the k-th segment of shear wall structure. The boundary conditions have the following form [7], [15], [18], [20]:

\[ N_{N,(1)}(0) = w, \quad w = -B^{-1}S_{E}z_0, \]
\[ N_{N,(k)}(h_k) = B_{N,(k+1)}^{-1}B_{N,k}N_{N,(k+1)}(h_k), \]
\[ N_{N,(k)}'(h_k) = N_{N,(k+1)}'(h_k), \]
\[ N_{N,(1)}'(H) = 0, \quad (2) \]

where \( S_{E} \) is \( n_n \times n_n \) boolean matrix, related to interaction between shear walls and continuous connections, \( z_0 \) is the vector containing given settlements of shear walls, \( h_k \) is the number of shear walls, \( h_k \) is the ordinate of k-th change of the cross-section and \( H \) is the structure height.

After determination of unknown functions of shear force intensity in continuous connections it is possible to obtain the function of horizontal displacements of the structure as well as its derivatives using the following equations:

\[ z \in (h_{k-1}, h_k) > V''_{N,(k)}(z) = V_{T,(k)}T_{E,(k)}(z) - V_{N,(k)}N_{N,(k)}(z), \quad (3) \]

where \( k \) is the index of a segment of the constant cross section, \( V(z) \) is a vector containing the functions of horizontal displacements of the structure, measured in the global ordinate system \( 0XYZ \) and \( T_{E,(k)}(z) \) is the vector of the functions of shear forces and a torque due to the action of lateral loads.

Matrices \( V_{T}, V_{N} \) appearing in the above relation are described by the following formulae:

\[ V_{T} = (L^T K_{T} L)^{-1}, \quad V_{N} = V_{T} L^T C N, \]
where \( L \) is \( 3n_h \times 3 \) matrix of coordinates transformation from the global coordinate system 0XYZ to the local systems, i.e. systems of principal axes of shear walls, \( K_{E} \) is \( 3n_e \times 3n_e \) matrix containing transverse stiffness of shear walls and \( C_{Y} \) is \( 3n_e \times n_e \) matrix containing the coordinates of the points of contraflexure in connections in the local systems of axes.

The boundary conditions have the following form:

\[
V_{(l)}(0) = 0, \quad V'_{(l)}(0) = 0, \quad V''_{(l)}(H) = 0. \tag{4}
\]

Besides, at the stations, where the cross sections of the walls change, the following compatibility conditions can be stated. From the geometric compatibility consideration we have:

\[
V_{(k)}(h_k) = V_{(k+1)}(h_k), \quad V'_{(k)}(h_k) = V'_{(k+1)}(h_k). \tag{5}
\]

From equilibrium consideration the following condition is obtained:

\[
m_{E(k)}(h_k) = m_{E(k+1)}(h_k), \tag{6}
\]

where \( m_{E}(z) \) is a vector of bending moments in shear walls, described by the relation:

\[
m_{E}(z) = K_{E}L V^{''}(z). \tag{7}
\]

Substituting (7) in Eqn (6) and next premultiplying by \( V_{n_0}L_{(k)/0} \), the following condition is obtained:

\[
V''_{(k)}(h_k) = S_{(k+1,k)} V''_{(k+1)}(h_k) \tag{8}
\]

where:

\[
S_{(k+1,k)} = V_{T(k)} L_{(k)}^T K_{Z(k+1)} L_{(k+1)^*}.
\]

3. Method of solution

In the proposed method the algorithm of solving the differential equation system, used for structures of constant cross-section [20], has been extended so as to enable us to take into account structures of the variable section. In order to uncouple differential equation systems auxiliary functions \( g_{0}(z) \) satisfying these relations have been introduced:

\[
N_{(k)}(z) = B_{(k)}^{1/2} Y_{(k)} G_{(k)}(z), \tag{9}
\]

where \( Y_{(k)} \) is matrix columns which are eigenvectors of the symmetric matrix \( P_{(k)} = B_{(k)}^{-1/2} A_{(k)} B_{(k)}^{1/2} \). Consequently, \( n_e \) second-order differential equations have been obtained in the following form:

\[
z \in (h_{k-1},h_k) > g_{0}^{''}(z) - \lambda_{i(k)} g_{0}(z) = F_{B_{(k)}}, \quad F_{B_{(k)}} = Y_{l(k)} B_{(k)}^{1/2} f_{l(k)}(z) \tag{10}
\]

where \( \lambda_{i(k)} \) is \( i \)-th eigenvalue of matrix \( P_{(k)} \), and \( Y_{l(k)} \) is eigenvector corresponding to the \( i \)-th eigenvalue. The eigenvalues and eigenvectors of symmetric matrix \( P_{(k)} \) are computed by a set of procedures realizing the Householder’s tridiagonalization and the QL algorithm, which have been inserted in Ref. [22] and later written in Pascal.

The form of solutions from Eqn (10) is as follows:

\[
g_{0}(z) = C_{l_{(k)}} e^{\sqrt{\lambda_{l_{(k)}}} z} + C_{2l_{(k)}} e^{-\sqrt{\lambda_{l_{(k)}}} z} + r_{0l_{(k)}} W_{2}(z), \tag{11}
\]

where \( C_{l_{(k)}}, C_{2l_{(k)}} \) are integration constants, \( r_{0l_{(k)}} \) are particular solution coefficients, calculated by indeterminate coefficient method and \( W_{2}(z) = \text{col} (z^2, \ldots, z^{n_e}) \).

Introducing Eqn (11) into the relation (9) and later considering boundary conditions (2) we will obtain the system of \( 2n_h n_e \) equations for the determination of all constants of integration in the form:

\[
R_{W} C = P_{S}, \tag{12}
\]

where \( R_{W} \) is unsymmetric matrix, \( C \) is a vector of integration constants and \( P_{S} \) is vector dependent on loadings. The solutions are computed by the procedures based on the LU factorization, where \( L \) is lower-triangular and \( U \) is upper-triangular, taken from Ref. [22].

The next step of computations is determining functions of horizontal displacements of the structure and their derivatives necessary to calculate internal forces and stresses.

The integration of functions \( V''(z) \) taking into consideration boundary condition \( V''_{(n_h)}(H) = 0 \) and the compatibility condition (8) yields the following expressions:

\[
z \in (h_{n_h-1},H) > V''_{(n_h)}(z) = \frac{\int_{H}^{z} V''_{(n_h)}(t) \, dt}{h_{n_h}}, \tag{13}
\]

\[
z \in (h_{k-1},h_k) > V''_{(k)}(z) = \frac{\int h_k^{z} V''_{(k)}(t) \, dt + S_{(k+1,k)} V''_{(k+1)}(h_k)}{h_k}. \tag{14}
\]

Next, integrating the above equations with regard to boundary conditions \( V_{(l)}(0) = 0, \quad V'_{(l)}(0) = 0 \) and compatibility conditions (5), the following is obtained:

\[
z \in (h_{k-1},h_k) > V_{(k)}(z) = \frac{\int_{h_{k-1}}^{z} V_{(k)}(t) \, dt + V_{(k-1)}(h_{k-1})}{h_{k-1}}, \tag{14}
\]

where: \( k = 1, \ldots, n_h \), \( h_0 = 0 \).

Integration is realized numerically.

On the basis of the presented algorithm the software included in the system for the analysis of shear wall tall buildings [20], [21] in the Delphi environment has been implemented.
4. Numerical examples

In the course of system testing there has been a good agreement of our results and those presented in Ref. [16], [14], [15], [6], [12], [3], [8] and obtained from tests on Araldite models [6]. To illustrate the correctness of algorithm realization, three examples of coupled shear walls of variable thickness have been chosen.

4.1. Example 1: Symmetrical shear wall with step change in thickness and uniform continuous connection

The 22-storey symmetrical coupled shear wall with a step change in thickness, previously studied by Rosman [15], is analysed. The storey height is 2.69 m, depth of walls is 6.50 m and span of continuous connections is 1.65 m. The shear wall thickness at the lower 10 storeys is 0.407 m and in the upper 12 storeys is 0.286 m. The floor slabs of depth 0.21 m and width 6.50 m are considered as continuous connections. The modulus of elasticity of concrete is taken to be \( E = 2.1 \times 10^5 \, \text{kG/cm}^2 \), and the shear modulus \( G = 3/7 \, E \). The wall is subjected to lateral load due to wind action. In Fig.1 there are diagrams of horizontal displacements and shear force intensity in continuous connection. The maximum displacement and maximum shear force intensity given in Ref. [15] are 0.0132 m and 5346 kG/m, respectively and it shows a good agreement.

4.2. Example 2: Asymmetrical shear wall with step change in thickness

In this example, analysed previously in Ref. [2], [3], the connecting beam as well as walls have step change in thickness. The 21-storey asymmetrical coupled shear wall consisted of two segments of different thickness, with a constant storey height of 1.0 m. All dimensions are given in inches. The shear wall thickness at the lower 11 storeys is 0.625 and in the upper 10 storeys is 0.375. The depth of the left and right wall is 3.0 and 2.5, respectively. The depth of connecting beams is 0.25. The effective span length of a beam is taken as \( 1.5 + 0.25 = 1.75 \). The adjustment to the span length of the spandrel beam is to allow for the fact that the rigid-end condition could not possibly occur immediately at the junction of the wall and the beam [13]. The shear wall is assumed to be made of isotropic material having Young’s modulus \( E \) of 463 000 lb/sq.in. and Poisson’s ratio of 0.0. The shear wall is subjected to a unit horizontal uniformly distributed load at the left side.

In Fig. 2 there is a plan of the shear wall and normal stress distribution across section at \( z = 3.375 \). The obtained diagrams of horizontal deflection and shear force intensity in continuous connection are shown in Fig. 3. The computations correlated well with the results obtained by the finite element method and the finite strip method [2], [3].

4.3. Example 3: Asymmetrical shear wall consisted of three segments of different thickness.

Fig. 4 shows the plan of 31-storey asymmetrical shear wall with two bands of openings created by the extension of Example 2. In the modified structure the wall of depth 2.5, connected by the same spandrel beams as in Example 2, has been inserted on the right side. Furthermore, the whole structure has been heighten by 10-storey segment of thickness 0.25. The properties of material and the loads are taken to be the same as in Example 2. Fig. 4 shows the normal stress distribution at the base of the structure. In Fig. 5 there are diagrams of horizontal displacements and shear force intensity in two continuous connections. The short time of computations for this example confirms the efficiency of the proposed algorithm.

5. Final remarks

The paper presents the algorithm for the analysis of shear wall structures of variable thickness, using a variant of the continuous connection method. The conducted tests have confirmed correctness of the algorithm realization. The proposed algorithm is effective and can be useful for a design analysis of tall buildings.

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References


Figure 1: Example 1 - Horizontal displacements and shear force intensity function in continuous connection
Figure 2: Example 2 - Plan of shear wall and normal stresses at z = 3.375

Figure 3: Example 2 - Horizontal displacements and shear force intensity function in continuous connection
Figure 4: Example 3 - Plan and normal stresses at the base of shear wall structure

Figure 5: Example 3 - Horizontal displacements and shear force intensity functions in two continuous connections