1 TASK

The gas temperature of a fully engulfed fire in an office has to be determined. The room of the “Simulated Office” test of the Cardington building is chosen for this analysis. The measured temperatures during the fully engulfed fire are shown in Figure 3, so the calculation can be compared with these results.

A natural fire model is chosen for the calculation of the gas temperature. For fires with a flash-over, the method of the compartment fires can be used. A simple calculation method for a parametric temperature-time curve is given in Annex A of EN 1991-1-2.

Figure 1. Cardington building (left) and the office of the “Simulated Office” test (right)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Floor area</td>
<td>$A_f = 135 , \text{m}^2$</td>
</tr>
<tr>
<td>Total area of enclosures</td>
<td>$A_t = 474 , \text{m}^2$</td>
</tr>
<tr>
<td>Total area of vertical openings</td>
<td>$A_v = 27 , \text{m}^2$</td>
</tr>
<tr>
<td>Vertical opening factor</td>
<td>$\alpha_v = 0.2$</td>
</tr>
<tr>
<td>Horizontal opening factor</td>
<td>$\alpha_h = 0.0$</td>
</tr>
<tr>
<td>Height</td>
<td>$H = 4.0 , \text{m}$</td>
</tr>
<tr>
<td>Average window height</td>
<td>$h_{eq} = 1.8 , \text{m}$ (assumption)</td>
</tr>
<tr>
<td>Lightweight concrete</td>
<td>$\rho = 1900 , \text{kg/m}^2$</td>
</tr>
<tr>
<td></td>
<td>$c = 840 , \text{J/kgK}$</td>
</tr>
<tr>
<td></td>
<td>$\lambda = 1.0 , \text{W/mK}$</td>
</tr>
</tbody>
</table>

Fire growth rate: medium
2 DETERMINATION OF FIRE LOAD DENSITY

For the determination of the fire load density the Annex E of EN 1991-1-2 offers a calculation model. The design value of the load density may either be given from a national fire load classification of occupancies and/or specific for an individual project by performing a fire load evaluation.

At this example, the second method is chosen.

\[
q_{f,d} = q_{f,k} \cdot m \cdot \delta_{q_1} \cdot \delta_{q_2} \cdot \delta_n
\]

where:
- \( m \) the combustion factor
- \( \delta_{q_1} \) the factor considering the danger of fire activation by size of the compartment
- \( \delta_{q_2} \) the factor considering the fire activation risk due to the type of occupancy
- \( \delta_n \) the factor considering the different active fire fighting measures

The fire load consisted of 20% plastics, 11% paper and 69% wood, so it consisted mainly of cellulosic material. Therefore the combustion factor is:

\[ m = 0.8 \]

The factor \( \delta_{q_1} \) considers the danger of fire activation by size of the compartment, as given in Table 1.

Table 1. Fire activation risk due to the size of the compartment (see EN 1991-1-2, Table E.1)

<table>
<thead>
<tr>
<th>Compartment floor area ( A_f ) [m²]</th>
<th>( \leq 25 )</th>
<th>( \leq 250 )</th>
<th>( \leq 2500 )</th>
<th>( \leq 5000 )</th>
<th>( \leq 10,000 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Danger of fire activation ( \delta_{q_1} )</td>
<td>1.10</td>
<td>1.50</td>
<td>1.90</td>
<td>2.00</td>
<td>2.13</td>
</tr>
</tbody>
</table>

\[ \delta_{q_1} = 1.5 \]

A factor \( \delta_{q_2} \) considers the fire activation risk due to the type of occupancy, as given in Table 2.

Table 2. Fire activation risk due to the type of occupancy (see EN 1991-1-2, Table E.1)

<table>
<thead>
<tr>
<th>Danger of fire activation ( \delta_{q_2} )</th>
<th>Examples of occupancies</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.78</td>
<td>artgallery, museum, swimming pool</td>
</tr>
<tr>
<td>1.00</td>
<td>offices, residence, hotel, paper industry</td>
</tr>
<tr>
<td>1.22</td>
<td>manufactory for machinery &amp; engines</td>
</tr>
<tr>
<td>1.44</td>
<td>chemical laboratory, painting workshop</td>
</tr>
<tr>
<td>1.66</td>
<td>manufactory for fireworks or paints</td>
</tr>
</tbody>
</table>

\[ \delta_{q_2} = 1.0 \]

The factor taking the different active fire fighting measures into account is calculated to:

\[ \delta_n = \prod_{i=1}^{10} \delta_{ni} \]

The factors \( \delta_{ni} \) are given in Table 3.
Table 3. Factors $\delta_{ni}$ (see EN 1991-1-2, Table E.2)

<table>
<thead>
<tr>
<th>Automatic fire suppression</th>
<th>Automatic water extinguishing system</th>
<th>$\delta_{n1}$</th>
<th>0.61</th>
</tr>
</thead>
<tbody>
<tr>
<td>Independent water supplies</td>
<td>$\delta_{n2}$</td>
<td>1</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.7</td>
<td></td>
</tr>
<tr>
<td>Automatic fire detection</td>
<td>Automatic fire detection &amp; alarm</td>
<td>$\delta_{n3}$</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>By heat or</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Automatic alarm transmission to fire brigade</td>
<td>$\delta_{n5}$</td>
<td>0.87</td>
</tr>
<tr>
<td>Manual fire suppression</td>
<td>Work Fire Brigade</td>
<td>$\delta_{n6}$</td>
<td>0.61</td>
</tr>
<tr>
<td></td>
<td>Off Site Fire Brigade</td>
<td>$\delta_{n7}$</td>
<td>0.78</td>
</tr>
</tbody>
</table>
|                            | Safe access routes                    | $\delta_{n8}$ | 0.9 or 1.0
|                            | Fire fighting devices                 | $\delta_{n9}$ | 1.0 or 1.5
|                            | Smoke exhaust system                  | $\delta_{n10}$ | 1.0 or 1.5

$\delta_n = 1.0\cdot0.73\cdot0.87\cdot0.78\cdot1.0\cdot1.0\cdot1.0 = 0.50$

For calculating the characteristic fire load, the characteristic fire load has to be determined. It is defined as:

$$Q_{fi,k} = \sum M_{k,i} \cdot H_{ui} \cdot \psi_i$$

where:

- $M_{k,i}$ the amount of combustible material [kg]
- $H_{ui}$ the net calorific value [MJ/kg], see EN 1991-1-2, Table E.3
- $\psi_i$ the optional factor for assessing protected fire loads

The total fire loading was equivalent to 46 kg wood/m², so the characteristic fire load is:

$$Q_{fi,k} = (135 \cdot 46) \cdot 17.5 \cdot 1.0 = 108,675 \text{ MJ}$$

The characteristic fire load density is determined to:

$$q_{f,k} = Q_{fi,k} / A_f = 108,675 / 135 = 805 \text{ MJ/m}^2$$

The design value of the fire load density is calculated to:

$$q_{f,d} = 805 \cdot 0.8 \cdot 1.5 \cdot 1.0 \cdot 0.5$$

$$= 483.0 \text{ MJ/m}^2$$

3 CALCULATION OF THE PARAMETRIC TEMPERATURE-TIME CURVE

It has to be determined if the fully engulfed fire is fuel or ventilation controlled. For this, the opening factor and the design value of the fire load density related to the total surface are needed.

$$O = \sqrt{h_{eq} \cdot A_i / A_f} = \sqrt{1.8 \cdot 27 / 474} = 0.076 \text{ m}^{1/2} \begin{cases} \geq 0.02 \\ \leq 0.2 \end{cases}$$
The determination if the fire is fuel or ventilation controlled is:

\[ 0.2 \cdot 10^{-3} \cdot q_{r,d} / O = 0.2 \cdot 10^{-3} \cdot 137.6 / 0.076 = 0.362 \text{ h} > t_{\text{lim}} = 0.333 \text{ h} \]

\[ \Rightarrow \text{The fire is ventilation controlled} \]

For calculation of the temperature-time curves for the heating and the cooling phase, the \( b \) factor is needed. This factor considers the thermal absorptivity for the boundary of enclosure. The density, the specific heat and the thermal conductivity of the boundary may be taken at ambient temperature. The floor, the slab and the walls are made of lightweight concrete:

\[ b = \sqrt{\rho \cdot c \cdot \lambda} = \sqrt{1900 \cdot 840 \cdot 1.0} = 1263.3 \frac{J}{m^2 \cdot s \cdot K} \]

\[ \geq 100 \]

\[ \leq 2200 \]

The temperature-time curve in the heating phase is given by:

\[ \theta_s = 20 + 1325 \cdot \left( 1 - 0.324 \cdot e^{-0.2 \cdot t^*} - 0.204 \cdot e^{-1.7 \cdot t^*} - 0.472 \cdot e^{-19 \cdot t^*} \right) \]

Because the fire is ventilation controlled, the time \( t^* \) is calculated to:

\[ t^* = t \cdot \Gamma \]

where:

\[ \Gamma = \frac{(O/b)^2}{(0.04/1160)^2} = \frac{(0.076/1263.3)^2}{(0.04/1160)^2} = 3.04 \]

Now the heating phase can be calculated:

\[ \theta_s = 20 + 1325 \cdot \left( 1 - 0.324 \cdot e^{-0.2 \cdot (3.04 \cdot t^*)} - 0.204 \cdot e^{-1.7 \cdot (3.04 \cdot t^*)} - 0.472 \cdot e^{-19 \cdot (3.04 \cdot t^*)} \right) \]

For calculation of the cooling phase, the maximum temperature is needed:

\[ \theta_{\text{max}} = 20 + 1325 \cdot \left( 1 - 0.324 \cdot e^{-0.2 \cdot t_{\text{max}}^*} - 0.204 \cdot e^{-1.7 \cdot t_{\text{max}}^*} - 0.472 \cdot e^{-19 \cdot t_{\text{max}}^*} \right) \]

where:

\[ t_{\text{max}}^* = t_{\text{max}} \cdot \Gamma \]

The time \( t_{\text{max}} \) is determined as below, where \( t_{\text{lim}} \) is given in Table 4.

\[ t_{\text{max}} = \max \left\{ 0.2 \cdot 10^{-3} \cdot q_{r,d} / O = 0.2 \cdot 10^{-3} \cdot 137.6 / 0.076 = 0.363 \text{ h} \right\} \]

\[ t_{\text{lim}} = 0.333 \text{ h} \]

<table>
<thead>
<tr>
<th>Table 4. Time ( t_{\text{lim}} ) for different fire growth rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slow fire growth rate</td>
</tr>
<tr>
<td>( t_{\text{lim}} ) [h]</td>
</tr>
</tbody>
</table>

So \( t_{\text{max}}^* \) is calculated to:

\[ t_{\text{max}}^* = 0.363 \cdot 3.04 = 1.10 \text{ h} \]
The maximum temperature is calculated to:
\[
\theta_{\text{max}} = 20 + 1325 \cdot \left(1 - 0.324 \cdot e^{-0.21^{1.10}} - 0.204 \cdot e^{-1.7^{1.10}} - 0.427 \cdot e^{-19.1^{1.10}}\right)
\]
\[= 958.8 \, ^\circ\text{C} \]

During the cooling phase, \(t^*\) and \(t_{\text{max}}^*\) are calculated to:
\[
t^* = t \cdot \Gamma = t \cdot 3.04 \, [\text{h}]
\]
\[
t_{\text{max}}^* = \left(0.2 \cdot 10^{-3} \cdot \frac{q_{\text{rd}}}{O}\right) \cdot \Gamma = 1.10 \text{h}
\]

The temperature-time curve in the cooling phase for \(0.5 \leq t_{\text{max}}^* \leq 2.0\) is given by:
\[
\theta_k = \theta_{\text{max}} - 250 \cdot \left(3 - t_{\text{max}}^* \right) \cdot \left(t^* - t_{\text{max}}^* \cdot x\right)
\]
\[= 958.8 - 250 \cdot (3 - 1.10) \cdot (t \cdot 3.04 - 1.10 \cdot 1.0)\]

where:
\[t_{\text{max}} > t_{\lim} \quad x = 1.0\]

Combination of the heating and cooling curves leads to the parametric temperature-time curve shown in Figure 2.

---

Figure 2. Gas temperature of the office calculated by using the parametric temperature-time curve of the office
4 COMPARISON BETWEEN CALCULATION AND FIRE TEST

To compare the calculation with the measured temperatures in the test, the factors $\delta_1$, $\delta_2$ and $\delta_n$ for calculation of the fire load density have to be set to 1.0 (see Figure 3).

![Figure 3. Comparison of measured and calculated temperature-time curves](image)

REFERENCES


The Behaviour of multi-storey steel framed buildings in fire, Moorgate: British Steel plc, Swinden Technology Centre, 1998

Valorisation Project: Natural Fire Safety Concept, Sponsored by ECSC, June 2001
The steel temperature of a beam has to be determined. It is part of an underground car park below the shopping mall Auchan in Luxembourg. The beams of the car park are accomplished without any use of fire protection material. The most severe fire scenario is a burning car in the middle of the beam (see Figure 1).

For getting the steel temperature, the natural fire model of a localised fire is used.

Figure 1. Underground car park of the shopping mall Auchan

Figure 2. Static system and cross-section of the beam
Diameter of the fire: \( D = 2.0 \text{ m} \)
Vertical distance between fire source and ceiling: \( H = 2.7 \text{ m} \)
Horizontal distance between beam and flame axis: \( r = 0.0 \text{ m} \)
Emissivity of the fire: \( \varepsilon_f = 1.0 \)
Configuration factor: \( \Phi = 1.0 \)
Stephan Boltzmann constant: \( \sigma = 5.67 \cdot 10^{-8} \text{ W/m}^2\text{K}^4 \)
Coefficient of the heat transfer: \( \alpha_c = 25.0 \text{ W/m}^2\text{K} \)

Steel profile: IPE 550
Section factor: \( A_m/V = 140 \text{ l/m} \)
Unit mass: \( \rho_a = 7850 \text{ kg/m}^3 \)
Surface emissivity: \( \varepsilon_m = 0.7 \)
Correction factor: \( k_{sh} = 1.0 \)

2 RATE OF HEAT RELEASE

The rate of heat release is normally determined by using the EN 1991-1-2 Section E.4. For dimensioning the beams at this car park, the rate of heat release for one car is taken from an ECSC project called "Development of design rules for steel structures subjected to natural fires in CLOSED CAR PARKS" (see Figure 3).

\[
L_f = -1.02 \cdot D + 0.0148 \cdot Q^{2/5} = -2.04 + 0.0148 \cdot Q^{2/5}
\]

3 CALCULATION OF THE STEEL TEMPERATURES

3.1 Calculation of the flame length

First of all, the flame length has to be determined.
A plot of this function with the values of Figure 3 is shown in Figure 4. With a ceiling height of 2.80 m, the flame is impacting the ceiling at a time from 16.9 min to 35.3 min (see Figure 4).

Figure 4. Flame length of the localised fire

It is important to know, if the flame is impacting the ceiling or not, because different calculation methods for the calculation of the net heat flux are used for these two cases (see Figure 5).

Figure 5. Flame models: Flame is not impacting the ceiling (A); Flame is impacting the ceiling (B)

3.2 Calculation of the net heat flux

3.2.1 1st case: The flame is not impacting the ceiling

The net heat flux is calculated according to Section 3.1 of EN 1991-1-2.

\[
\dot{\Phi}_{net} = \dot{\alpha} \cdot (\theta_c - \theta_m) + \Phi \cdot \varepsilon_m \cdot \varepsilon_f \cdot \sigma \cdot \left( \left( \theta_c + 273 \right)^4 - \left( \theta_m + 273 \right)^4 \right)
\]

\[
= 25.0 \cdot (\theta_c - \theta_m) + 3.969 \cdot 10^{-8} \cdot \left( \left( \theta_c + 273 \right)^4 - \left( \theta_m + 273 \right)^4 \right)
\]

Section 3.1
The gas temperature is calculated to:

\[
\theta_{(c)} = 20 + 0.25 \cdot (0.8 \cdot Q)^{2/3} \cdot (z - z_0)^{-5/3} \leq 900 \, ^\circ C
\]

where:
- \(z\) is the height along the flame axis (2.7 m)
- \(z_0\) is the virtual origin of the axis [m]

\[
z_0 = -1.02 \cdot D + 0.0052 \cdot Q^{2/5} = -2.04 + 0.0052 \cdot Q^{2/5}
\]

3.2.2 2nd case: The flame is impacting the ceiling

The net heat flux, if the flame is impacting the ceiling, is given by:

\[
\hat{h}_{\text{net}} = \hat{h}_c - \alpha_c \cdot (\theta_m - 20) - \Phi \cdot \epsilon_m \cdot \epsilon_f \cdot \sigma \cdot (\vartheta_m + 273)^4 - (293)^4
\]

\[
= \hat{h}_c - 25.0 \cdot (\theta_m - 20) - 3.969 \cdot 10^{-8} \cdot (\vartheta_m + 273)^4 - (293)^4
\]

The heat flux depends on the parameter \(y\). For different dimensions of \(y\), different equations for determination of the heat flux have to be used.

if \(y \leq 0.30\):

\[
\hat{h}_c = 100,000
\]

if \(0.30 < y < 1.0\):

\[
\hat{h}_c = 136,300 - 121,000 \cdot y
\]

if \(y \geq 1.0\):

\[
\hat{h}_c = 15,000 \cdot y^{3.7}
\]

where:

\[
y = \frac{r + H + z'}{L_n + H + z'} = \frac{2.7 + z'}{L_n + 2.7 + z'}
\]

The horizontal flame length is calculated to:

\[
L_n = \left(2.9 \cdot H \cdot \left(Q_{H'}\right)^{0.33}\right) - H = \left(7.83 \cdot \left(Q_{H'}\right)^{0.33}\right) - 2.7
\]

where:

\[
Q_{H'} = \frac{Q}{\left(1.11 \cdot 10^6 \cdot H^{2.5}\right)} = \frac{Q}{\left(1.11 \cdot 10^6 \cdot 2.7^{2.5}\right)}
\]

The vertical position of the virtual heat source is determined to:

if \(Q_{D'} < 1.0\): \(z' = 2.4 \cdot D \cdot \left(Q_{D'}^{2/5} - \left(Q_{D'}\right)^{2/5}\right) = 4.8 \cdot \left(\left(Q_{D'}\right)^{2/5} - \left(Q_{D'}\right)^{2/5}\right)\)

if \(Q_{D'} \geq 1.0\): \(z' = 2.4 \cdot D \cdot \left(1.0 - \left(Q_{D'}^{2/5}\right) = 4.8 \cdot \left(1.0 - \left(Q_{D'}\right)^{2/5}\right)\)

where:

\[
Q_{D'} = \frac{Q}{\left(1.11 \cdot 10^6 \cdot D^{2.5}\right)} = \frac{Q}{\left(1.11 \cdot 10^6 \cdot 2.0^{2.5}\right)}
\]
3.3 Calculation of the steel temperature-time curve

The specific heat of the steel $c_a$ is needed to calculate the steel temperature. The parameter is given by EN 1993-1-2, Section 3.4.1.2 depending on the steel temperature.

$$\theta_{a,j} = \theta_m + k_m \cdot \frac{A_w}{V} \cdot \frac{\rho}{\rho_{net}} \cdot \Delta t = \theta_m + \frac{1.78 \cdot 10^{-3}}{c_a} \cdot \Delta t$$

The steel temperature-time curve is shown in Figure 6. Additionally, the results of the FEM-analysis done by PROFILARBED are shown for comparison.

![Figure 6. Specific heat of carbon steel (see EN 1993 Part 1-2, Figure 3.4)](image1)

REFERENCES


ECSC Project, Development of design rules for steel structures subjected to natural fires in CLOSED CAR PARKS, CEC agreement 7210-SA/211/318/518/620/933, Brussels, June 1996
Example to EN 1993 Part 1-2: Column with axial loads

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1 TASK

In the following example, a column of a department store will be dimensioned for fire resistance. The column is part of a braced frame and is connected bending resistant to the upper and lower column. The length is 3.0 m. During fire exposure, the buckling length can be reduced as seen below in Figure 1. The loads are centric axial compression forces. The column is exposed to fire on four sides. A hollow encasement of gypsum is chosen for fire protection. The required standard fire resistance class for the column is R 90.

![Figure 1. Buckling lengths of columns in braced frames](image)

Figure 1. Buckling lengths of columns in braced frames

![Figure 2. Cross-section of the column](image)

Figure 2. Cross-section of the column

Material properties:
Column:
- Profile: rolled section HE 300 B
- Steel grade: S 235
- Cross-section class: 1
Yield stress: \( f_y = 23.5 \text{kN/cm}^2 \)
Cross-sectional area: \( A_a = 149 \text{cm}^2 \)
Elastic modulus: \( E_a = 21,000 \text{kN/cm}^2 \)
Moment of inertia: \( I_a = 8560 \text{cm}^4 \) (weak axis)

Encasement:
- Material: gypsum
- Thickness: \( d_p = 3.0 \text{cm} \) (hollow encasement)
- Thermal conductivity: \( \lambda_p = 0.2 \text{W/(m·K)} \)
- Specific heat: \( c_p = 1700 \text{J/(kg·K)} \)
- Density: \( \rho_p = 945 \text{kg/m}^3 \)

Loads:
- Permanent actions: \( G_k = 1200 \text{kN} \)
- Variable actions: \( P_k = 600 \text{kN} \)

2. FIRE RESISTANCE OF COLUMN

2.1 Mechanical actions during fire exposure

The accidental situation is used for the combination of mechanical actions during fire exposure.

\[
E_{dk} = E \left( \sum G_k + A_d + \sum \psi_{2,i} \cdot Q_{k,i} \right)
\]

The combination factor for department stores is \( \psi_{2,1} = 0.6 \). So the axial load is determined to:

\[
N_{pd} = 1200 + 0.6 \cdot 600 = 1560 \text{kN}
\]

2.2 Calculation of the maximum steel temperature

The analysis of EN 1993-1-2 is used to calculate the steel temperature of the hollow encased column. For a hollow encased member, the section factor is calculated to:

\[
\frac{A_p}{V} = 2 \cdot (b + h)/A_p = 2 \cdot (30 + 30) \cdot 10^2 /149 = 81 \text{ m}^{-1}
\]

By using the Euro-Nomogram (ECCS No.89), the maximal temperature \( \theta_{a,max,90} \) of the steel bar is:

\[
\left( \frac{A_p}{V} \right) \left( \frac{\lambda_p}{d_p} \right) = 81 \cdot 0.2/0.03 = 540 \text{W/m}^3\text{K}
\]

\[
\Rightarrow \theta_{a,max,90} \approx 445 \text{ °C}
\]

2.3 Verification in the temperature domain

Within EN 1993-1-2 the verification in the temperature domain is not allowed for members in which stability phenomena have to be taken into account.

2.4 Verification in the strength domain

The verification in the strength domain during fire exposure is carried out as a plastic ultimate state of the load-carrying capacity.

\[
E_{f,i,d} \leq R_{f,i,d}
\]
In this example, the verification has to be done with the axial forces.

\[ N_{f,d} \leq N_{b,f,i,Rd} \]

The design resistance under high temperature conditions is calculated as:

\[ N_{b,f,i,Rd} = \chi_{fi} \cdot A \cdot k_{y,\theta,\max} \cdot \frac{f_y}{\gamma_{M,fi}} \]

In dependence of \( \theta_{a,\max,90} \) the reduction factors \( k_{y,\theta} \) and \( k_{E,\theta} \) are given in Table 3.1 of the EN 1993-1-2. For intermediate values of the steel temperature, linear interpolation may be used.

\[ k_{y,445^\circ C} = 0.901 \]
\[ k_{E,445^\circ C} = 0.655 \]

The load-carrying capacity is determined in consideration of the non-dimensional slenderness during fire exposure.

\[ \chi_{fi} = \chi \cdot \sqrt{k_{y,\theta}/k_{E,\theta}} = 0.21 \cdot \sqrt{0.901/0.655} = 0.25 \]

where:

\[ \chi = L_{K_u}/(i \cdot \lambda_a) = (0.5\cdot 300)/(7.58\cdot 93.9) = 0.21 \]

With the non-dimensional slenderness the reduction factor for flexural buckling \( \chi_{fi,\theta} \) can be calculated.

\[ \chi_{fi} = \frac{1}{\varphi_b + \sqrt{\varphi_b^2 - \chi^2}} = \frac{1}{0.61 + \sqrt{0.61^2 - 0.25^2}} = 0.86 \]

where:

\[ \varphi = 0.5 \cdot [1 + \alpha \cdot \chi + \chi^2] = 0.5 \cdot [1 + 0.65 \cdot 0.25 + 0.25^2] = 0.61 \]

and:

\[ \alpha = 0.65 \cdot \sqrt{235/f_y} = 0.65 \cdot \sqrt{235/235} = 0.65 \]

The design resistance arises to:

\[ N_{b,f,i,Rd} = 0.86 \cdot 149 \cdot 0.901 \cdot \frac{23.5}{1.0} = 2713 \text{ kN} \]

Verification:

\[ N_{f,d}/N_{b,f,i,Rd} = 1560/2713 = 0.58 < 1 \quad \checkmark \]

REFERENCES


Example to EN 1993 Part 1-2: Beam with bending and compression loads

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1 TASK

This example deals with a beam subjected to a uniform load, which causes a bending moment, and an axial load. Stability phenomena have to be considered. The beam is part of an office building. A hollow encasement of gypsum is chosen for fire protection. Due to a concrete slab the beam is exposed to fire on three sides. There is no shear-connection between the beam and the slab. The required standard fire resistance class for the beam is R 90.

![Figure 1. Static system](image1)

![Figure 2. Beam Cross-section](image2)

Material properties:

Beam:
- Profile: rolled section HE 200 B
- Steel grade: S 235
- Cross-section class: 1
- Yield stress: $f_y = 235$ N/mm²
- Elastic modulus: $E = 210,000$ N/mm²
- Shear modulus: $G = 81,000$ N/mm²
- Cross-sectional area: $A_a = 7810$ mm²
- Moment of inertia: $I_z = 2000$ cm⁴
Torsion constant: \( I_t = 59.3 \text{ cm}^4 \)
Warping constant: \( I_w = 171,100 \text{ cm}^6 \)
Section moduli:
\[ W_{el,v} = 570 \text{ cm}^2 \]
\[ W_{pl,v} = 642.5 \text{ cm}^3 \]
Encasement:
Material: gypsum
Thickness: \( d_p = 20 \text{ mm (hollow encasement)} \)
Thermal conductivity: \( \lambda_p = 0.2 \text{ W/(m-K)} \)
Specific heat: \( c_p = 1700 \text{ J/(kg-K)} \)
Density: \( \rho_p = 945 \text{ kg/m}^3 \)
Loads:
Permanent Loads: \( G_k = 96.3 \text{ kN} \)
\( g_k = 1.5 \text{ kN/m} \)
Variable Loads: \( p_k = 1.5 \text{ kN/m} \)

2 FIRE RESISTANCE OF BEAM WITH BENDING AND COMPRESSION LOADS

2.1 Mechanical actions during fire exposure

The combination of mechanical actions during fire exposure shall be calculated as an accidental situation:
\[ E_{dh} = E \left( \sum G_k + A_j + \sum \psi_{2,i} \cdot Q_{i,j} \right) \]

The combination factor for office buildings is \( \psi_{2,1} = 0.3 \). The design loads under high temperature conditions are:
\[ N_{f,d} = 96.3 \text{ kN} \]
\[ M_{f,d} = \left[ 1.5 + 0.3 \cdot 1.5 \right] \cdot \frac{10.0^2}{8} = 24.38 \text{ kNm} \]

2.2 Calculation of steel temperatures

The steel temperature is given by the Euro-Nomogram (ECCS No.89). Therefore the section factor \( A_p/V \) is needed. For a hollow encased member exposed to fire on three sides, the section factor is:
\[ \frac{A_p}{V} = \frac{2 \cdot h + b}{A_s} = \frac{2 \cdot 20.0 + 20.0}{78.1} \cdot 10^3 = 77 \text{ m}^{-1} \]

With
\[ \frac{A_p \cdot \lambda_p}{V \cdot d_p} = 77 \cdot \frac{0.2}{0.02} = 770 \frac{W}{\text{m}^3 \cdot \text{K}} \]

the critical temperature arises to:
\[ \Rightarrow \theta_{a,\text{max},90} \approx 540 \text{ °C} \]
2.3 Verification in the temperature domain

Due to section 4.2.4 (2) of EN 1993-1-2 the verification in the temperature domain may not be accomplished, because of stability problems of the beam.

2.4 Verification in the strength domain

Members with a Class 1 cross-section should be analysed for the problem of flexural buckling and of lateral torsional buckling.

2.4.1 Flexural buckling

The verification for flexural buckling is:

\[ \frac{N_{f.d}}{\chi_{\min,fi} \cdot A \cdot k_{\chi,\theta} \cdot f_y / \gamma_{M,fi}} + \frac{k_{y,\theta} \cdot M_{f.d}}{W_{pl,y} \cdot k_{y,\theta} \cdot f_y / \gamma_{M,fi}} \leq 1 \]

The reduction factor \( \chi_{\min,fi} \) is the minimum of the two reduction factors for flexural buckling \( \chi_{y,fi} \) and \( \chi_{z,fi} \). The non-dimensional slenderness for the temperature \( \theta_a \) is needed for the calculation of these reduction factors.

For calculation of the non-dimensional slenderness in the fire situation, the non-dimensional slenderness at ambient temperatures have to be determined.

\[ \bar{\lambda}_y = \frac{L_{cr}}{i_y \cdot \lambda_a} = \frac{1000}{8.54 \cdot 93.9} = 1.25 \]
\[ \bar{\lambda}_z = \frac{L_{cr}}{i_z \cdot \lambda_a} = \frac{1000}{5.07 \cdot 93.9} = 2.10 \]

The needed reduction factors \( k_{y,\theta} \) and \( k_{E,\theta} \) are given in EN 1993-1-2 Table 3.1:

\[ k_{y,\theta} = 0.656 \]
\[ k_{E,\theta} = 0.484 \]

With the reduction factors, the non-dimensional slenderness in the fire situation can be determined:

\[ \bar{\lambda}_{y,\theta} = \bar{\lambda}_y \sqrt{\frac{k_{y,\theta}}{k_{E,\theta}}} = 1.25 \sqrt{\frac{0.656}{0.484}} = 1.46 \]
\[ \bar{\lambda}_{z,\theta} = \bar{\lambda}_z \sqrt{\frac{k_{y,\theta}}{k_{E,\theta}}} = 2.1 \sqrt{\frac{0.656}{0.484}} = 2.44 \]

With
\[ \alpha = 0.65 \cdot \sqrt{235 / f_y} = 0.65 \cdot \sqrt{235 / 235} = 0.65 \]

and
\[ \varphi_{y,\theta} = \frac{1}{2} \left( 1 + \alpha \cdot \bar{\lambda}_{y,\theta} + \bar{\lambda}_{y,\theta}^2 \right) = \frac{1}{2} \left( 1 + 0.65 \cdot 1.46 + 1.46^2 \right) = 2.04 , \]
\[ \varphi_{z,\theta} = \frac{1}{2} \left( 1 + \alpha \cdot \bar{\lambda}_{z,\theta} + \bar{\lambda}_{z,\theta}^2 \right) = \frac{1}{2} \left( 1 + 0.65 \cdot 2.44 + 2.44^2 \right) = 4.27 \]
the reduction factors $\chi_{y,\beta}$ and $\chi_{z,\beta}$ can be calculated:

$$\chi_{y,\beta} = \frac{1}{\phi_{y,\theta} + \sqrt{\phi_{y,\theta}^2 - \lambda_{y,\theta}^2}} = \frac{1}{\frac{2.04}{2.04^2 - 1.46^2}} = 0.29$$

$$\chi_{z,\beta} = \frac{1}{\phi_{z,\theta} + \sqrt{\phi_{z,\theta}^2 - \lambda_{z,\theta}^2}} = \frac{1}{\frac{4.27}{4.27^2 - 2.44^2}} = 0.13$$

Verification:

$$\frac{96.3}{0.13 \cdot 78.1 \cdot 0.656 \cdot 23.5} + \frac{1.50 \cdot 2438}{642.5 \cdot 0.656 \cdot 23.5} = 0.98 < 1 \quad \checkmark$$

where:

$$\mu_y = (1.2 \cdot \beta_{M,y} - 3) \cdot \lambda_{y,\theta} + 0.44 \cdot \beta_{M,y} - 0.29$$

$$= (1.2 \cdot 1.3 - 3) \cdot 1.46 + 0.44 \cdot 1.3 - 0.29$$

$$= -1.82$$

$$k_y = 1 - \frac{\mu_y \cdot N_{y,d}}{\chi_{y,\beta} \cdot A_k \cdot k_{y,\theta} \cdot f_y / \gamma_{M,\beta}} = 1 - \frac{-1.82 \cdot 96.3}{0.29 \cdot 78.1 \cdot 0.656 \cdot 23.5 / 1.0} = 1.50$$

### 2.4.2 Lateral torsional buckling

The second verification deals with the problem of lateral torsional buckling.

$$\frac{N_{y,d}}{\chi_{y,\beta} \cdot A_k \cdot k_{y,\theta} \cdot f_y / \gamma_{M,\beta}} + \frac{k_{LT} \cdot M_{y,d}}{\chi_{LT,\beta} \cdot W_{pl,y} \cdot k_{y,\theta} \cdot f_y / \gamma_{M,\beta}} \leq 1$$

For calculation of the non-dimensional slenderness in the fire situation, the non-dimensional slenderness at ambient temperatures has to be determined.

$$\bar{\lambda}_{LT} = \sqrt{\frac{642.5 \cdot 23.5}{14,203.5}} = 1.03$$

where:

$$M_{cr} = \zeta \cdot \frac{\pi^2 \cdot E \cdot I_z}{2} \cdot \left( \sqrt{c^2 + 0.25 \cdot z_p^2} + 0.5 \cdot z_p \right)$$

$$= 11.2 \cdot \frac{\pi^2 \cdot 21,000 \cdot 2000}{(1.0 \cdot 1000)^2} \cdot \left( \sqrt{1241.9 + 0.25 \cdot \left( \frac{20}{2} \right)^2} - 0.5 \cdot \frac{20}{2} \right)$$

$$= 14,203.5 \text{ kNcm}$$

with:

$$c^2 = \frac{I_z + 0.039 \cdot I_T}{I_T} = \frac{171,100 + 0.039 \cdot 1000^2}{1000} = 1241.9$$

During fire exposure, the non-dimensional slenderness changes to:

$$\bar{\lambda}_{LT,\theta} = \bar{\lambda}_{LT} \cdot \sqrt{\frac{k_{x,\theta}}{k_{E,\theta}}} = 1.03 \cdot \sqrt{\frac{0.656}{0.484}} = 1.20$$

with
\[
\phi_{LT,\theta} = \frac{1}{2}\left(1 + \alpha \cdot \phi_{z,\theta} + \lambda_{LT,\theta} \cdot \phi_{z,\theta}^2 \right) = \frac{1}{2}\left(1 + 0.65 \cdot 1.20 + 1.20^2 \right) = 1.61,
\]

the reduction factor \( \chi_{LT,\theta} \) is calculated to:

\[
\chi_{LT,\theta} = \frac{1}{\phi_{LT,\theta} + \sqrt{\phi_{LT,\theta}^2 - \phi_{z,\theta}^2}} = \frac{1}{1.61 + \sqrt{1.61^2 - 1.20^2}} = 0.37
\]

Verification:

\[
\begin{align*}
\frac{96.3}{0.13 \cdot 78.1 - 0.65 \cdot 23.5} + \frac{0.80 \cdot 2438}{0.37 \cdot 642.5 - 0.65 \cdot 23.5/1.0} &= 0.62 + 0.53 = 1.15 \leq 1
\end{align*}
\]

where:

\[
\begin{align*}
k_{LT} &= 1 - \frac{\mu_{LT} \cdot N_{f,d}}{\chi_{z,\theta} \cdot A \cdot k_{z,\theta} \cdot f_y} = 1 - \frac{0.33 \cdot 96.3}{0.13 \cdot 78.1 \cdot 0.656 \cdot 23.5} = 0.80
\end{align*}
\]

\[
\begin{align*}
\mu_{LT} &= 0.15 \cdot \lambda_{c,\theta} \cdot \beta_{M,LT} - 0.15 < 0.9 \\
&= 0.15 \cdot 2.44 \cdot 1.3 - 0.15 \\
&= 0.33 < 0.9
\end{align*}
\]

REFERENCES

DIN 18800, Stahlbauten, Teil 2 Stabilitätsfälle, Knicken bei Stäben, Berlin: Beuth Verlag GmbH, Germany, November 1990

ECCS No.89, Euro-Nomogram, Brussels: ECCS – Technical Committee 3 – Fire Safety of Steel Structures, 1995


Literatur for MCr (for example: Steel Construction Manual)
1 TASK

At this example, a beam made of a welded hollow section has to be dimensioned. It is part of a hall roof structure. The length of the beam is 35.0 m and the beams are arranged at a distance of 10.0 m. It is charged with uniform loads and is restrained against lateral evasion. The beam is executed without any use of fire protection material. The required standard fire resistance class for the tensile bar is R 30.

Figure 1. Static system

Figure 2. Cross-section

Material properties:

Steel grade: S 355
Yield stress: \( f_y = 355 \text{ N/mm}^2 \)
Height: \( h = 700 \text{ mm} \)
Height of web: \( h_w = 650 \text{ mm} \)
Width: \( b = 450 \text{ mm} \)
Thickness of flange: \( t_f = 25 \text{ mm} \)
Thickness of web: $t_w = 25$ mm
Cross-sectional area of the flange: $A_f = 11,250$ mm²
Cross-sectional area of the web: $A_w = 16,250$ mm²
Specific heat: $c_a = 600$ J/(kg·K)
Density: $\rho_a = 7850$ kg/m³
Emissivity of the beam: $\varepsilon_m = 0.7$
Emissivity of the fire: $\varepsilon_r = 1.0$
Configuration factor $\Phi = 1.0$
Coefficient of the heat transfer: $\alpha_c = 25.0$ W/m²K
Stephan Boltzmann constant: $\sigma = 5.67 \cdot 10^{-8}$ W/m²K⁴

Loads:
- Permanent actions:
  - Beam: $g_{a,k} = 4.32$ kN/m
  - Roof: $g_{r,k} = 5.0$ kN/m
- Variable actions:
  - Snow: $p_{s,k} = 11.25$ kN/m

2 FIRE RESISTANCE OF BEAM MADE OF A HOLLOW SECTION

2.1 Mechanical actions during fire exposure
The accidental situation is used for the combination of mechanical actions during fire exposure.

$$E_{nh} = E \left( \sum G_k + A_j + \sum \psi_{2,i} \cdot Q_{k,i} \right)$$

The combination factor for snow loads is $\psi_{2,1} = 0.0$. With this parameter, the design bending load is calculated to:

$$M_{f,n} = \left[ (4.32 + 5.0) + 0.0 \cdot 11.25 \right] \cdot \frac{35.0^2}{8} = 1427.1$$ kNm

2.2 Calculation of the steel temperature
The temperature increase of the steel section is calculated to:

$$\Delta \theta_{a,i} = k_{sh} \cdot \frac{A_n \cdot V}{c_a \cdot \rho_a} \cdot k_{net,d} \cdot \Delta t = 1.0 \cdot \frac{40}{600 \cdot 7850} \cdot 5 \cdot k_{net} = 4.25 \cdot 10^{-5} \cdot k_{net}$$

where:
- $k_{sh}$ correction factor for the shadow effect ($k_{sh} = 1.0$)
- $\Delta t$ time interval ($\Delta t = 5$ seconds)
- $A_n/V$ section factor for the unprotected beam (Table 4.2)

The net heat flux is calculated according to EN 1991 Part 1-2.

$$k_{net} = \alpha_c \cdot (\theta_f - \theta_m) + \Phi \cdot \varepsilon_m \cdot \varepsilon_f \cdot \sigma \cdot \left( (\theta_m + 273)^4 - (\theta_m + 273)^4 \right)$$

The standard temperature-time curve is used for getting the gas temperatures.

$$\theta_f = 20 + 345 \cdot \log_{10} (8 \cdot t + 1)$$
The steel temperature-time curve of the hollow section is shown in Figure 3:

![Steel temperature-time curve](image)

Figure 3. Steel temperature-time curve of the hollow section

\[ \theta_{a,\text{max},30} = 646 \, ^\circ\text{C} \]

### 2.3 Verification in the temperature domain

The design moment resistance during fire exposure at the time \( t = 0 \) is needed to get the utilization factor.

\[ M_{\text{fl, Rd}, 0} = W_{pl} \cdot f_y \cdot k_{y,\theta,\text{max}} \cdot \gamma_{M,\text{fl}} \]

\[ = 12,875,000 \cdot 355 \cdot \frac{1.0}{1.0} \cdot 10^{-6} \]

\[ = 4570.6 \, \text{kNm} \]

where:

\[ k_{y,\theta,\text{max}} = 1.0 \quad \text{for} \, \theta = 20 \, ^\circ\text{C} \text{ at the time } t = 0 \]

\[ \gamma_{M,\text{fl}} = 1.0 \]

and:

\[ W_{pl} = 2 \cdot \left( \frac{2 \cdot A_w \cdot h_w}{4} + A_f \cdot \frac{h - t_w}{2} \right) \]

\[ = 2 \cdot \left( 16,250 \cdot \frac{650}{4} + 11,250 \cdot \frac{700 - 25}{2} \right) \]

\[ = 12,875,000 \, \text{mm}^3 \]

The utilization factor is calculated to:

\[ \mu_0 = E_{f, d} \cdot R_{f, d, 0} = M_{f, d} \cdot \frac{M_{\text{fl, Rd}, 0}}{1427.1/4570.6} = 0.31 \]

The critical temperature \( \theta_{a,cr} \) is given in Table 4.1 of the EN 1993 Part 1-2.

\[ \Rightarrow \theta_{a,cr} = 659 \, ^\circ\text{C} \]

Verification:

\[ \frac{646}{659} = 0.98 < 1 \quad \checkmark \]
2.4 Verification in the strength domain

To calculate the moment resistance the reduction factor $k_{y,\theta}$ has to be determined for the temperature $\theta_{a,max,30} = 646 \, ^\circ\text{C}$. This factor is given in Table 3.1 of the EN 1993 Part 1-2:

$$k_{y,\theta} = 0.360$$

Additionally, the adaptation factors $\kappa_1$ and $\kappa_2$ have to be determined.

The adaptation factor $\kappa_1$ considers the non-uniform temperature distribution across the cross-section.

Table 1. Adaptation factor $\kappa_1$

<table>
<thead>
<tr>
<th>Description</th>
<th>$\kappa_1$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam exposed on all four sides</td>
<td>1.0</td>
</tr>
<tr>
<td>Unprotected beam exposed on three sides with a composite or concrete slab on side four</td>
<td>0.7</td>
</tr>
<tr>
<td>Protected beam exposed on three sides with a composite or concrete slab on side four</td>
<td>0.85</td>
</tr>
</tbody>
</table>

The beam in this is an unprotected beam exposed to fire on four sides. Therefore $\kappa_1$ is set to:

$$\kappa_1 = 1.0$$

The adaptation factor $\kappa_2$ considers the non-uniform temperature distribution along a beam.

Table 2. Adaptation factor $\kappa_2$

<table>
<thead>
<tr>
<th>Description</th>
<th>$\kappa_2$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>At the supports of a statically indeterminate beam</td>
<td>0.85</td>
</tr>
<tr>
<td>In all other cases</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The verification is done in the middle of the beam and it is statically determinate. So the adaptation factor $\kappa_2$ is set to:

$$\kappa_2 = 1.0$$

Therefore the design moment resistance is calculated to:

$$M_{f,i,Rd} = M_{pl,Rd,20^\circ\text{C}} \cdot k_{y,\theta} \cdot \frac{\gamma_{M,1}}{\gamma_{M,\beta}} \cdot \frac{1}{\kappa_1 \cdot \kappa_2}$$

$$= (12,875,000 \cdot 355/1.1) \cdot 0.36 \cdot \frac{1.1}{1.0} \cdot \frac{1}{1.0 \cdot 1.0} \cdot 10^6 = 1645.4 \, \text{kNm}$$

Verification:

$$\frac{1427.1}{1645.4} = 0.87 < 1 \quad \checkmark$$

REFERENCES

Example to EN 1994 Part 1-2: Composite slab

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1 TASK

A composite slab has to be dimensioned in the fire situation. It is part of a shopping centre and the span is 4.8 m. The slab will be dimensioned as a series of simply supported beams. The required standard fire resistance class for the slab is R 90.

Figure 1. Static system

Figure 2. Steel sheet

Material properties

Steel sheet:
Yield stress: \( f_{yp} = 350 \text{ N/mm}^2 \)
Cross-sectional area: \( A_p = 1562 \text{ mm}^2/\text{m} \)
Parameters for m+k method: \( k = 0.150 \text{ N/mm}^2 \)

Concrete:
Strength category: C 25/30
Compression strength: \( f_c = 25 \text{ N/mm}^2 \)
Height: \( h_c = 140 \text{ mm} \)
Cross-sectional area: \( A_c = 131,600 \text{ mm}^2/\text{m} \)
Loads:
Permanent loads:
  Steel sheet \( g_{p,k} = 0.13 \text{kN/m}^2 \)
  Concrete \( g_{c,k} = 3.29 \text{kN/m}^2 \)
  Finishing load \( g_{f,k} = 1.2 \text{kN/m}^2 \)
Variable loads:
  Live load \( p_k = 5.0 \text{kN/m}^2 \)

Design sagging moment at ambient temperatures: \( M_{s,d} = 39.56 \text{kNm} \)

2 FIRE RESISTANCE OF A COMPOSITE SLAB

The composite slab has to be verified according to Section 4.3 and Annex D.

2.1 Geometrical parameters and scope of application

![Geometry of cross-section](image)

Table 1. Scope of application for slabs made of normal concrete and re-entrant steel sheets

<table>
<thead>
<tr>
<th>Scope of application for re-entrant profiles [mm]</th>
<th>Existing geometrical parameters [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>77.0 (\leq l_1 \leq 135.0 )</td>
<td>( l_1 = 115.0 )</td>
</tr>
<tr>
<td>110 (\leq l_2 \leq 150.0 )</td>
<td>( l_2 = 140.0 )</td>
</tr>
<tr>
<td>38.5 (\leq l_3 \leq 97.5 )</td>
<td>( l_3 = 38.0 )</td>
</tr>
<tr>
<td>50.0 (\leq h_1 \leq 130.0 )</td>
<td>( h_1 = 89.0 )</td>
</tr>
<tr>
<td>30.0 (\leq h_2 \leq 70.0 )</td>
<td>( h_2 = 51.0 )</td>
</tr>
</tbody>
</table>

2.2 Mechanical actions during fire exposure

The load is determined by the combination rule for accidental situations.

\[
E_{da} = E \left( \sum G_k + A_d + \sum \psi_{2,i} \cdot Q_k \right)
\]

According to EN 1994 Part 1-2, the load \( E_d \) may be reduced by the reduction factor \( \eta_{fd} \). It is calculated to:

\[
\eta_{fd} = \frac{G_k + \psi_{2,i} \cdot Q_k}{\gamma_G \cdot G_k + \gamma_{Q,i} \cdot Q_k} = \frac{0.13 + 3.29 + 1.2 + 0.6 \cdot 5.0}{1.35 \cdot (0.13 + 3.29 + 1.2) + 1.5 \cdot 5.0} = 0.55
\]

With \( \eta_{fd} \), the design bending moment \( M_{fd,d} \) can be calculated:

\[
M_{fd,d} = \eta_{fd} \cdot M_s = 0.55 \cdot 39.56 = 21.76 \text{kNm/m}
\]
2.3 Thermal insulation

The thermal insulation criteria “I” has to ensure the limitation of the thermal condition of the member. The temperature on top of the slab should not exceed 140 °C in average and 180 °C at its maximum.

The verification is done in the time domain. The time in which the slab fulfils the criteria “I” is calculated to:

\[ t_i = a_0 + a_1 \cdot h_1 + a_2 \cdot \Phi + a_3 \cdot \frac{A}{L_r} + a_4 \cdot \frac{1}{l_3} + a_5 \cdot \frac{A}{L_r} \cdot \frac{1}{l_3} \]

The rib geometry factor \( \frac{A}{L_r} \) is equivalent to the section factor \( \frac{A_p}{V} \) for beams. The factor considers that the mass and height have positive effects on the heating of the slab.

The view factor \( \Phi \) considers the shadow effect of the rib on the upper flange.

\[ \Phi = \left[ \sqrt{h_z^2 + \left( \frac{l_1 - l_2}{2} \right)^2} - \sqrt{h_z^2 + \left( \frac{l_1 - l_2}{2} \right)^2} \right] / l_3 \]

\[ = \left[ \sqrt{51^2 + \left( \frac{38 + 115 - 140}{2} \right)^2} - \sqrt{51^2 + \left( \frac{115 - 140}{2} \right)^2} \right] / 38 \]

\[ = 0.119 \]

The coefficients \( a_i \) for normal weight concrete is given in Table 2:

<table>
<thead>
<tr>
<th>Material</th>
<th>( a_0 )</th>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( a_3 )</th>
<th>( a_4 )</th>
<th>( a_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal weight concrete</td>
<td>-28.8</td>
<td>1.55</td>
<td>-12.6</td>
<td>0.33</td>
<td>-735</td>
<td>48.0</td>
</tr>
<tr>
<td>Light weight concrete</td>
<td>-79.2</td>
<td>2.18</td>
<td>-2.44</td>
<td>0.56</td>
<td>-542</td>
<td>52.3</td>
</tr>
</tbody>
</table>

With these parameters, \( t_i \) is calculated to:

\[ t_i = (-28.8) + 1.55 \cdot 89 + (-12.6) \cdot 0.119 \]

\[ + 0.33 \cdot 27 + (-735) \cdot 1/38 + 48 \cdot 27 \cdot 1/38 \]

\[ = 131.48 \text{ min} > 90 \text{ min} \]
2.4 Verification of the load carrying-capacity

The plastic moment design resistance is calculated to:

\[ M_{f,i,\text{pl}} = \sum A_i \cdot z_i \cdot k_{y,i} \cdot \left( \frac{f_{y,i}}{Y_{M,fi}} \right) + \alpha_{\text{lab}} \cdot \sum A_j \cdot z_j \cdot k_{c,j} \cdot \left( \frac{f_{c,j}}{Y_{M,fc}} \right) \]

To get the reduction factors \( k_{y,\theta} \) for the upper flange, lower flange and the web, the temperatures have to be determined. These are calculated to:

\[ \theta_a = b_0 + b_1 \cdot \frac{1}{l_3} + b_2 \cdot A + b_3 \cdot \Phi + b_4 \cdot \Phi^2 \]

The coefficients \( b_i \) can be obtained from Table 3:

<table>
<thead>
<tr>
<th>Concrete</th>
<th>Fire resistance [min]</th>
<th>Part of steel sheet</th>
<th>( b_0 ) [°C]</th>
<th>( b_1 ) [°C·mm]</th>
<th>( b_2 ) [°C/mm]</th>
<th>( b_3 ) [°C]</th>
<th>( b_4 ) [°C]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal weight concrete</td>
<td>60</td>
<td>Lower flange</td>
<td>951</td>
<td>-1197</td>
<td>-2.32</td>
<td>86.4</td>
<td>-150.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Web</td>
<td>661</td>
<td>-833</td>
<td>-2.96</td>
<td>537.7</td>
<td>-351.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Upper flange</td>
<td>340</td>
<td>-3269</td>
<td>-2.62</td>
<td>1148.4</td>
<td>-679.8</td>
</tr>
<tr>
<td></td>
<td>90</td>
<td>Lower flange</td>
<td>1018</td>
<td>-839</td>
<td>-1.55</td>
<td>65.1</td>
<td>-108.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Web</td>
<td>816</td>
<td>-959</td>
<td>-2.21</td>
<td>464.9</td>
<td>-340.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Upper flange</td>
<td>618</td>
<td>-2786</td>
<td>-1.79</td>
<td>767.9</td>
<td>-472.0</td>
</tr>
<tr>
<td></td>
<td>120</td>
<td>Lower flange</td>
<td>1063</td>
<td>-679</td>
<td>-1.13</td>
<td>46.7</td>
<td>-82.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Web</td>
<td>925</td>
<td>-949</td>
<td>-1.82</td>
<td>344.2</td>
<td>-267.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Upper flange</td>
<td>770</td>
<td>-2460</td>
<td>-1.67</td>
<td>592.6</td>
<td>-379.0</td>
</tr>
</tbody>
</table>

For the different parts of the steel sheet, the temperatures are:

Lower flange:

\[ \theta_{a,lf} = 1018 - 839 \cdot \frac{1}{38} - 1.55 \cdot 27 + 65.1 \cdot 0.119 - 108.1 \cdot 0.119^2 \]

\[ = 960.29 \text{ °C} \]

Web:

\[ \theta_{a,w} = 816 - 959 \cdot \frac{1}{38} - 2.21 \cdot 27 + 464.9 \cdot 0.119 - 340.2 \cdot 0.119^2 \]

\[ = 781.60 \text{ °C} \]

Upper flange:

\[ \theta_{a,uf} = 618 - 2786 \cdot \frac{1}{38} - 1.79 \cdot 27 + 767.9 \cdot 0.119 - 472.0 \cdot 0.119^2 \]

\[ = 580.87 \text{ °C} \]

To get the required load carrying-capacity during fire exposure, reinforcing bars have to be installed which normally are neglected for the ambient
temperature design. For each rib, one reinforcing bar Ø 10 mm is chosen. The position of the bar can be seen in Figure 5.

![Figure 5. Arrangement of the reinforcing bar](image)

The temperature of the reinforcing bar is calculated to:

$$\theta = c_0 + c_1 \cdot \frac{u_1}{h_2} + c_2 \cdot z + c_3 \cdot \frac{A}{L} + c_4 \cdot \alpha + c_5 \cdot \frac{1}{l_3}$$

where:

$$\frac{1}{z} = \frac{1}{\sqrt{u_1}} + \frac{1}{\sqrt{u_2}} + \frac{1}{\sqrt{u_3}}$$

$$= \frac{1}{\sqrt{\frac{l_1}{2}}} + \frac{1}{\sqrt{\frac{l_2}{2}}} + \frac{1}{\sqrt{h_2 + 10}}$$

$$= \frac{1}{\sqrt{57}} + \frac{1}{\sqrt{57}} + \frac{1}{\sqrt{61}}$$

$$= 0.393 \text{ mm}^{0.5}$$

$$\Rightarrow z = 2.54 \text{ mm}^{0.5}$$

![Figure 6. Definition of the distances $u_1$, $u_2$, $u_3$ and the angle $\alpha$](image)

The coefficients $c_i$ for normal weight concrete is given in Table 4.

<table>
<thead>
<tr>
<th>Concrete</th>
<th>Fire resistance [min]</th>
<th>$c_0$ [°C]</th>
<th>$c_1$ [°C]</th>
<th>$c_2$ [°C/mm$^{0.5}$]</th>
<th>$c_3$ [°C/mm]</th>
<th>$c_4$ [°C/°]</th>
<th>$c_5$ [°C]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal weight concrete</td>
<td>60</td>
<td>1191</td>
<td>-250</td>
<td>-240</td>
<td>-5.01</td>
<td>1.04</td>
<td>-925</td>
</tr>
<tr>
<td>Normal weight concrete</td>
<td>90</td>
<td>1342</td>
<td>-256</td>
<td>-235</td>
<td>-5.30</td>
<td>1.39</td>
<td>-1267</td>
</tr>
<tr>
<td>Normal weight concrete</td>
<td>120</td>
<td>1387</td>
<td>-238</td>
<td>-227</td>
<td>-4.79</td>
<td>1.68</td>
<td>-1326</td>
</tr>
</tbody>
</table>

With these parameters, the temperature of the reinforcing bar is:
\[
\theta_i = 1342 + (-256) \cdot \frac{61}{51} + (-235) \cdot 2.54 \\
+ (-5.30) \cdot 27 + 1.39 \cdot 104 + (-1267) \cdot \frac{1}{38}
\]
\[
= 407.0 \, ^\circ \text{C}
\]

For the steel sheet, the reduction factors \( k_{y,i} \) are given in Table 3.2 of the EN 1994-1-2. For the reinforcement the reduction factor is given in Table 3.4, because the reinforcement bars are cold worked.

The carrying-capacity for each part of the steel sheet and the reinforcing bars can now be calculated.

Table 5. Reduction factors and carrying-capacities

<table>
<thead>
<tr>
<th>Temperature ( \theta_i ) [°C]</th>
<th>Reduction factor ( k_{y,i} ) [-]</th>
<th>Partial area ( A_i ) [cm²]</th>
<th>( f_{y,i} ) [kN/cm²]</th>
<th>( Z_i ) [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower flange 960.29</td>
<td>0.047</td>
<td>1.204</td>
<td>35.0</td>
<td>1.98</td>
</tr>
<tr>
<td>Web 781.60</td>
<td>0.132</td>
<td>0.904</td>
<td>35.0</td>
<td>4.18</td>
</tr>
<tr>
<td>Upper flange 580.87</td>
<td>0.529</td>
<td>0.327</td>
<td>35.0</td>
<td>6.05</td>
</tr>
<tr>
<td>Reinforcement 407.0</td>
<td>0.921</td>
<td>0.79</td>
<td>50.0</td>
<td>36.38</td>
</tr>
</tbody>
</table>

The plastic neutral axis is calculated as equilibrium of the horizontal forces. The equilibrium is set up for one rib \( b = l_1 + l_2 \).

\[
z_{pl} = \sum \frac{Z_i}{a_{slab} \cdot (l_1 + l_3) \cdot f_c} = 1.98 + 4.18 + 6.05 + 36.38 \\
= \frac{0.85 \cdot (115 + 38) \cdot 25 \cdot 10^{-3}}{15.0} = 15.0 \, \text{mm}
\]

The plastic moment resistance for one rib is determined to:

Table 6. Calculation of the moment resistance of one rib

| Lower flange | 1.98 | 14.0 | 27.72 |
| Web | 4.18 | 14.0 - 5.1 / 2 = 11.45 | 47.86 |
| Upper flange | 6.05 | 14.0 - 5.1 = 8.9 | 53.85 |
| Reinforcement | 36.38 | 14.0 - 5.1 - 1.0 = 7.9 | 287.4 |
| Concrete | -48.59 | 1.50 / 2 = 0.75 | -36.44 |
| \( \sum \) 380.39 |

With the plastic moment of \( M_{pl,rib} = 3.80 \, \text{kNm} \) and the width \( w_{rib} = 0.152 \, \text{m} \) of one rib, the plastic moment resistance of the composite slab is:

\[
M_{f_i,d} = \frac{3.80}{0.152} = 25.00 \, \text{kNm/m}
\]

Verification:

\[
\frac{21.76}{25.00} = 0.88 < 1 \quad \checkmark
\]

REFERENCES


1 TASK

A fire safety verification has to be done for a composite beam of an office building. It is a simply supported beam and is loaded uniformly. The concrete slab of the composite beam protects the beam from the top in the fire situation, so the steel beam is exposed to fire on three sides. For fire protection of the steel beam a contour encasement of plaster is chosen. The required standard fire resistance class for the beam is R 60.

Material properties:

Beam:
- Profile: rolled section HE 160 B
- Steel grade: S 355
- Height: \( h = 160 \text{ mm} \)
- Height of web: \( h_w = 134 \text{ mm} \)
- Width: \( b = b_1 = b_2 = 160 \text{ mm} \)
Thickness of web: \( e_w = 8 \text{ mm} \)
Thickness of flange: \( e_f = e_1 = e_2 = 13 \text{ mm} \)
Cross-sectional area: \( A_a = 5430 \text{ mm}^2 \)
Yield stress: \( f_{y,a} = 355 \text{ N/mm}^2 \)

Slab:
- Strength category: C 25/30
- Height: \( h_c = 160 \text{ mm} \)
- Effective width: \( b_{eff} = 1400 \text{ mm} \)
- Compression strength: \( f_c = 25 \text{ N/mm}^2 \)
- Elastic modulus: \( E_{cm} = 29,000 \text{ N/mm}^2 \)

Shear connectors:
- Quantity: \( n = 34 \) (equidistant)
- Diameter: \( d = 22 \text{ mm} \)
- Tensile strength: \( f_u = 500 \text{ N/mm}^2 \)

Encasement:
- Material: plaster
- Thickness: \( d_p = 15 \text{ mm} \) (contour encasement)
- Thermal conductivity: \( \lambda_p = 0.12 \text{ W/(m·K)} \)
- Specific heat: \( c_p = 1100 \text{ J/(kg·K)} \)
- Density: \( \rho_p = 550 \text{ kg/m}^3 \)

Loads:
- Permanent loads:
  - Self weight: \( g_k = 20.5 \text{ kN/m} \)
  - Finishing load: \( g_k = 7.5 \text{ kN/m} \)
- Variable loads:
  - Live load: \( p_k = 15.0 \text{ kN/m} \)

2 FIRE RESISTANCE OF A COMPOSITE BEAM

2.1 Mechanical actions during fire exposure

Actions on structures from fire exposure are classified as accidental situation:

\[
E_{da} = E \left( \sum G_k + A_d + \sum \psi_{2,1} \cdot Q_k \right)
\]

The partial safety factor \( \gamma_{GA} \) for the accidental situation is \( \gamma_{GA} = 1.0 \). The combination factor for the leading variable action for office buildings is set to \( \psi_{2,1} = 0.3 \).

With these parameters, the design bending moment during fire exposure can be calculated:

\[
M_{f,i} = (20.5 + 7.5) + 0.3 \cdot (15.0) \cdot \frac{5.6^2}{8} = 127.4 \text{ kNm}
\]

2.2 Calculation of the temperatures in the cross-section

For the calculation of the temperatures, the cross-section is split into different sections. These are the concrete slab, the upper flange, the web and the lower flange. It is done according to Section 4.3.4.2 of EN 1994-1-2.

The temperatures of the upper flange, the web and the lower flange are determined by using the Euro-Nomogram ("Euro-Nomogram", ECCS No.89, 1996). Therefore, the section factors of these parts are required.
Lower flange:
\[
\frac{A_p}{V_l} = 2 \cdot (h_t + e_t) \cdot \frac{b_t \cdot e_t}{0.16 \cdot 0.013} = 166.3 \text{ m}^{-1}
\]

Web:
\[
\frac{A_p}{V_w} = 2 \cdot (h_w) \cdot \frac{b_w \cdot e_w}{0.134 \cdot 0.008} = 250.0 \text{ m}^{-1}
\]

Upper flange (more than 85% of the upper flange is in contact with the concrete slab):
\[
\frac{A_p}{V_u} = \frac{b_u + 2 \cdot e_2}{b_2 \cdot e_2} \cdot \frac{(0.16 + 2 \cdot 0.013)}{0.16 \cdot 0.013} = 89.4 \text{ m}^{-1}
\]

The temperatures are determined to:

Table 1. Temperatures of upper flange, web and lower flange

<table>
<thead>
<tr>
<th>Section 4.3.4.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{A_p}{V} ), ( \frac{d_p \cdot \lambda_p}{W} )</td>
</tr>
<tr>
<td>Upper flange</td>
</tr>
<tr>
<td>Web</td>
</tr>
<tr>
<td>Lower flange</td>
</tr>
</tbody>
</table>

The temperature of the concrete slab is not constant over its thickness. Therefore the compression strength varies over the thickness. For temperatures lower than 250 °C, the compression strength is not reduced. Above 250 °C it is reduced by the reduction factor \( k_{c,\theta} \). Assessment of the temperatures may be done in layers of 10 mm thickness on basis of Table 2.

Table 2. Temperature distribution in a solid slab of 100 mm thickness composed of normal weight concrete and not insulated (see EN 1994-1-2, Annex D.3, Table D.5)

<table>
<thead>
<tr>
<th>Depth ( x ) [mm]</th>
<th>Temperature ( q_c ) [°C] after a fire duration in min. of 30’ 60’ 90’ 120’ 180’ 240’</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>535</td>
</tr>
<tr>
<td>10</td>
<td>470</td>
</tr>
<tr>
<td>15</td>
<td>415</td>
</tr>
<tr>
<td>20</td>
<td>350</td>
</tr>
<tr>
<td>25</td>
<td>300</td>
</tr>
<tr>
<td>30</td>
<td>250</td>
</tr>
<tr>
<td>35</td>
<td>210</td>
</tr>
<tr>
<td>40</td>
<td>180</td>
</tr>
<tr>
<td>45</td>
<td>160</td>
</tr>
<tr>
<td>50</td>
<td>140</td>
</tr>
<tr>
<td>55</td>
<td>125</td>
</tr>
<tr>
<td>60</td>
<td>110</td>
</tr>
<tr>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>100</td>
<td>60</td>
</tr>
</tbody>
</table>
2.3 Verification using simple calculation model

The composite beam is verified by the simple calculation model. It is done in the strength domain. The calculation of the moment resistance is accomplished according to Annex E.

The temperatures of the parts of the steel beam were determined in Section 3.2. The reduction factors $k_{y,\theta,i}$ for the calculation of the yield stresses at elevated temperatures, are given in Table 3.2 of EN 1994-1-2, Section 3.2.1.

<table>
<thead>
<tr>
<th>$\theta_{\text{max},i0} , [^\circ\text{C}]$</th>
<th>$k_{y,\theta} [-]$</th>
<th>$f_{y,i,\theta} , [\text{kN/cm}^2]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper flange</td>
<td>390</td>
<td>1.00</td>
</tr>
<tr>
<td>Web</td>
<td>650</td>
<td>(0.47 + 0.23)/2 = 0.35</td>
</tr>
<tr>
<td>Lower flange</td>
<td>550</td>
<td>(0.78 + 0.47)/2 = 0.625</td>
</tr>
</tbody>
</table>

The next step is the calculation of the tensile force $T$ of the steel beam according to Figure 3.

$$T = f_{y,i,\theta 1} \cdot (b \cdot e_f) + f_{y,i,\theta u} \cdot (h_u \cdot e_u) + f_{y,i,\theta 2} \cdot (b \cdot e_f) \cdot \gamma_{M,fi,a}$$

$$= \frac{22.2 \cdot (16 \cdot 1.3) + 12.4 \cdot (13.4 \cdot 0.8) + 35.5 \cdot (16 \cdot 1.3)}{1.0}$$

$$= 1333.1 \, \text{kN}$$

The location of the tensile force is determined to:

$$y_T = \frac{f_{y,i,\theta 1} \cdot \left( \frac{e_f^2}{2} \right) + f_{y,i,\theta u} \cdot (h_u \cdot e_u) \cdot \left( \frac{e_f}{2} + \frac{h_u}{2} \right) + f_{y,i,\theta 2} \cdot (b \cdot e_f) \left( h - \frac{e_f}{2} \right)}{T \cdot \gamma_{M,fi,a}}$$

$$= \frac{22.2 \cdot \left( \frac{1.3^2}{2} \right) + 12.4 \cdot (13.4 \cdot 0.8) \cdot \left( 1.3 + \frac{13.4}{2} \right) + 35.5 \cdot (16 \cdot 1.3) \cdot \left( 16 - \frac{1.3}{2} \right)}{1333.1 \cdot 1.0}$$

$$= 9.53 \, \text{cm}$$
In a simply supported beam, the value of the tensile force $T$ is limited by:

$$T \leq N \cdot P_{fi,Rd}$$

where:
- $N$ Number of shear connectors in one of the critical lengths of the beam
- $P_{fi,Rd}$ Design resistance in the fire situation of a shear connector

To get $P_{fi,Rd}$, the reduction factors $k_{u,\theta}$ and $k_{c,\theta}$ (Table 5) as well as the design resistances of a shear connector at ambient temperatures $P_{Rd,1}$ and $P_{Rd,2}$ are needed.

The temperatures for getting the reduction factors are determined as 80% (stud connector) and 40% (concrete) of the steel flange (see EN 1994 Part 1-2, Section 4.3.4.2.5 (2)). The reduction factor for the tensile strength of the stud connector is given in Table 3.3 of EN 1994-1-2, Section 3.2.1. The effect of strain hardening ($k_{u,\theta} > 1$) should only be accounted if it is proven that local failures (i.e. local buckling, shear failure, concrete spalling, etc) do not occur. So in this case the strain hardening is not considered. The reduction factor for the compression strength of the concrete is given in Table 3.3 of EN 1994-1-2, Section 3.2.1.

$$\theta_v = 0.8 \cdot 390 = 312 \, ^\circ C$$

$$\Rightarrow k_{u,\theta} = 1.0$$

$$\theta_c = 0.4 \cdot 390 = 156 \, ^\circ C$$

$$\Rightarrow k_{c,\theta} = 0.98$$

The design resistances of the shear connector are calculated according to EN 1994-1-1, with the partial safety factor $\gamma_{M,fi,v}$ replacing $\gamma_v$.

$$P_{Rd,1} = 0.8 \cdot \frac{f_y \cdot \pi \cdot d^2}{\gamma_{M,fi,v} \cdot 4} = 0.8 \cdot \frac{50.0 \cdot \pi \cdot 2.2^2}{1.0 \cdot 4} = 152 \, kN$$

$$P_{Rd,2} = 0.29 \cdot \alpha \cdot d^2 \cdot \sqrt{\frac{f_y \cdot E_{cm}}{\gamma_{M,fi,v}}} = 0.29 \cdot 1.0 \cdot 2.2^2 \cdot \sqrt{\frac{2.5 \cdot 2900}{1.0}} = 120 \, kN$$

The design resistance in the fire situation of a shear connector is:

$$P_{fi,Rd} = \min \begin{cases} P_{fi,Rd,1} = 0.8 \cdot k_{u,\theta} \cdot P_{Rd,1} = 0.8 \cdot 1.0 \cdot 152 = 121.6 \, kN \\ P_{fi,Rd,2} = k_{c,\theta} \cdot P_{Rd,2} = 0.98 \cdot 120 = 117.6 \, kN \quad \leftarrow \text{relevant} \end{cases}$$

So, the limitation can be verified:

$$1333.1 \, kN < 34/2 \cdot 117.6 = 1999.2 \, kN$$

For equilibrium of forces, the compression force has to be equal to the tension force. Therefore the thickness of the compressive zone $h_u$ is determined to:

$$h_u = \frac{T}{b_{ef} \cdot f_c / \gamma_{M,fi,c}} = \frac{1333.1}{140.0 \cdot 2.5/1.0} = 3.8 \, cm$$

Now, two situations may occur. The first one is that the temperature in every layer of the concrete in the compression zone is lower than 250 °C. In the second situation the temperature of some layers of the concrete is above 250 °C. To check which situation occurs, following calculation has to be done:

$$\left( h_c - h_u \right) = 16 - 3.8 = 12.2 \, cm$$
If the result of this equation is greater than the depth $x$ according to Table 2, corresponding to a concrete temperature below 250 °C, the concrete in the compression zone may not be reduced.

$$h_{cr} = x = 5.0 \text{ cm} < 12.2 \text{ cm}$$

The point of application of the compression force $y_F$ is determined to:

$$y_F = h + h_c - \left( \frac{h_u}{2} \right) = 16 + 16 - \left( \frac{3.8}{2} \right) = 30.1 \text{ cm}$$

The moment resistance is calculated to:

$$M_{f_{cr}, Rd} = T \cdot (y_F - y_T) = 1333.1 \cdot (30.1 - 9.53) \cdot 10^{-2} = 274.2 \text{ kNm}$$

Verification:

$$\frac{127.4}{274.2} = 0.46 < 1 \quad \checkmark$$

REFERENCES

ECCS No.89, Euro-Nomogram, Brussels: ECCS – Technical Committee 3 – Fire Safety of Steel Structures, 1995
1 TASK

A fire safety verification has to be done for a composite beam of a storehouse. It is a simply supported beam with a uniform load and has a span of 12.0 m. The steel beam is partially encased and the slab is a composite slab with a re-entrant steel sheet. The required standard fire resistance class for the beam is R 90.

Figure 1. Static system

Figure 2. Cross-section
Material properties:

Beam:
- Profile: rolled section IPE 500
- Steel grade: S 355
- Height: \( h = 500 \text{ mm} \)
- Width: \( b = 200 \text{ mm} \)
- Thickness of web: \( e_w = 10.2 \text{ mm} \)
- Thickness of flange: \( e_f = 16 \text{ mm} \)
- Cross-sectional area: \( A_a = 11,600 \text{ mm}^2 \)
- Yield stress: \( f_{y,a} = 355 \text{ N/mm}^2 \)

Slab:
- Strength category: C 25/30
- Height: \( h_c = 160 \text{ mm} \)
- Effective width: \( b_{eff} = 3000 \text{ mm} \)
- Compression strength: \( f_c = 25 \text{ N/mm}^2 \)

Profiled steel sheet:
- Type: re-entrant
- Height: \( h_a = 51 \text{ mm} \)

Reinforcement in partial concrete encasement:
- Steel grade: S 500
- Diameter: \( 2 \text{ Ø 30} \)
- Cross-sectional area: \( A_s = 1410 \text{ mm}^2 \)
- Axis distances: \( u_1 = 110 \text{ mm} \)
- \( u_{1,1} = 60 \text{ mm} \)
- Yield stress: \( f_{y,s} = 500 \text{ N/mm}^2 \)

Concrete between flanges:
- Strength category: C 25/30
- Width: \( b_c = 200 \text{ mm} \)
- Compression strength: \( f_c = 25 \text{ N/mm}^2 \)

Loads:
- Permanent loads:
  - Self-weight: \( g_{s,k} = 15.0 \text{ kN/m} \)
  - Finishing load: \( g_{f,k} = 6.0 \text{ kN/m} \)
- Variable loads:
  - Live load: \( p_k = 30.0 \text{ kN/m} \)

2. FIRE RESISTANCE OF A COMPOSITE BEAM COMPRISING STEEL BEAM WITH PARTIAL CONCRETE ENCASEMENT

2.1 Mechanical actions during fire exposure

Actions on structures in fire situation are classified as an accidental situation:

\[
E_{dk} = E \left( \sum G_k + A_d + \sum \psi_{2,k} Q_{k} \right)
\]

The combination factor for the leading variable action and for a storehouse is \( \psi_{2,1} = 0.8 \).

With these parameters, the design bending moment during fire exposure can be calculated:

\[
M_{\beta,d} = \left( (15.0 + 6.0) + 0.8 \cdot (30.0) \right) \frac{12.0^2}{8} = 810.0 \text{ kNm}
\]
2.2 Verification using simple calculation model

The composite beam is verified by the simple calculation model. It is accomplished according to EN 1994 Part 1-2, Section 4.3.4.3 and Annex F.

To use this model, the slab should have a minimum thickness $h_c$. Additionally the steel beam should have a minimum height $h$, a minimum width $b_c$ (where $b_c$ is the minimum width of steel beam or concrete encasement) and a minimum area $h \cdot b_c$ (see Table 1).

Table 1. Minimum dimensions for the use of the simple calculation model for composite beams comprising steel beams with partial concrete encasement (see EN 1994 Part 1-2, Section 4.3.4.3, Table 4.8)

<table>
<thead>
<tr>
<th>Standard fire resistance</th>
<th>Minimum slab thickness $h_c$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>R 30</td>
<td>60</td>
</tr>
<tr>
<td>R 60</td>
<td>80</td>
</tr>
<tr>
<td>R 90</td>
<td>100</td>
</tr>
<tr>
<td>R 120</td>
<td>120</td>
</tr>
<tr>
<td>R 180</td>
<td>150</td>
</tr>
</tbody>
</table>

$h_c = 160 \text{ mm} > \min h_c = 100 \text{ mm}$

In the calculation model of Annex F, the cross section is divided into different parts. At some parts the yield stress at other parts the cross-sectional area is reduced.

Figure 3. Reduced cross-section for the calculation of the plastic moment resistance and stress distributions in steel (A) and concrete (B)

The heating of the concrete slab is considered by reducing the cross-sectional area. For the different fire resistance classes, the thickness reduction $h_{c,fi}$ is given in Table 2. For composite slabs made with re-entrant steel sheets, a minimum thickness reduction $h_{c,fi,min}$ has to be considered. This minimum thickness reduction is equal to the height of the steel sheet (see EN 1994 Part 1-2, Annex F, Figure F.2).

$h_{c,fi} = 30 \text{ mm}$
For this, the height of the concrete during fire exposure $h_{c,h}$ is:

$$h_{c,h} = 160 - 51 = 109 \text{ mm}$$

<table>
<thead>
<tr>
<th>Standard fire resistance</th>
<th>Slab reduction $h_{c,i}$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>R 30</td>
<td>10</td>
</tr>
<tr>
<td>R 60</td>
<td>20</td>
</tr>
<tr>
<td>R 90</td>
<td>30</td>
</tr>
<tr>
<td>R 120</td>
<td>40</td>
</tr>
<tr>
<td>R 180</td>
<td>55</td>
</tr>
</tbody>
</table>

The heating of the upper flange of the steel beam is considered by reducing its cross-sectional area. The calculation of the width reduction $b_{fi}$ is shown in Table 3.

$$b_{fi} = (16.0/2) + 30 + (200 - 200)/2 = 38.0 \text{ mm}$$

The effective width is calculated to:

$$b_{fi,u} = 200 - 2 \cdot 38 = 124.0 \text{ mm}$$

The web of the steel beam is divided into two parts. The upper part of the web possesses the full yield stress, where the yield stress of the lower part has a linear gradient, from the yield stress of the upper part to the reduced yield stress of the lower flange. The height of the lower part of the web $h_l$ is calculated to:

$$h_l = \frac{a_e}{b_l} \cdot \frac{a_s \cdot e_n}{b_l \cdot h} > h_{l,\text{min}}$$
The parameters \( a_1 \) and \( a_2 \), as well as the minimum height \( h_{\text{min}} \), are given in Table 4 for \( h/b_c > 2 \).

\[
h/b_c = 500 \text{ mm} / 200 \text{ mm} = 2.5
\]

\[
l_h = \frac{14,000}{200} + \frac{75,000 \cdot 10.2}{200 \cdot 500} = 77.7 \text{ mm} > 40 \text{ mm}
\]

Table 4. Parameters \( a_1 \), \( a_2 \) and minimum height \( h_{\text{min}} \) for \( h/b_c > 2 \) (see EN 1994 Part 1-2, Annex F, Table F.3)

<table>
<thead>
<tr>
<th>Standard fire resistance</th>
<th>( a_1 ) [mm²]</th>
<th>( a_2 ) [mm²]</th>
<th>( h_{\text{min}} ) [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>R 30</td>
<td>3600</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>R 60</td>
<td>9500</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>R 90</td>
<td>14,000</td>
<td>75,000</td>
<td>40</td>
</tr>
<tr>
<td>R 120</td>
<td>23,000</td>
<td>110,000</td>
<td>45</td>
</tr>
<tr>
<td>R 180</td>
<td>35,000</td>
<td>250,000</td>
<td>55</td>
</tr>
</tbody>
</table>

The lower flange is not reduced by its cross-sectional area. Here, the yield stress is reduced by the factor \( k_a \). This factor is limited by a minimum and maximum value. These limits, as well as the calculation of \( k_a \), are given in Table 5.

\[
a_0 = 0.018 \cdot e_f + 0.7 = 0.018 \cdot 16.0 + 0.7 = 0.988
\]

\[
k_a = \begin{cases} 
0.12 - \frac{17}{200} + \frac{500}{38 \cdot 200} \cdot 0.988 = 0.100 & (> 0.06) \\
0.12 & (< 0.12)
\end{cases}
\]

Table 5. Reduction factor \( k_a \) of the yield stress of the lower flange (see EN 1994 Part 1-2, Annex F, Table F.4)

<table>
<thead>
<tr>
<th>Standard fire resistance</th>
<th>Reduction factor ( k_a )</th>
<th>( k_{a,m} )</th>
<th>( k_{a,max} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>R 30</td>
<td>( 1.12 - \frac{84}{h/b_c} + \frac{h}{22 \cdot b_c} ) ( a_0 )</td>
<td>0.5</td>
<td>0.8</td>
</tr>
<tr>
<td>R 60</td>
<td>( 0.21 - \frac{26}{h/b_c} + \frac{h}{24 \cdot b_c} ) ( a_0 )</td>
<td>0.12</td>
<td>0.4</td>
</tr>
<tr>
<td>R 90</td>
<td>( 0.12 - \frac{17}{h/b_c} + \frac{h}{38 \cdot b_c} ) ( a_0 )</td>
<td>0.06</td>
<td>0.12</td>
</tr>
<tr>
<td>R 120</td>
<td>( 0.1 - \frac{15}{h/b_c} + \frac{h}{40 \cdot b_c} ) ( a_0 )</td>
<td>0.05</td>
<td>0.1</td>
</tr>
<tr>
<td>R 180</td>
<td>( 0.03 - \frac{3}{h/b_c} + \frac{h}{50 \cdot b_c} ) ( a_0 )</td>
<td>0.03</td>
<td>0.06</td>
</tr>
</tbody>
</table>

The heating of the reinforcing bars in the partial concrete encasement is considered by reducing the yield stress. The reduction factor depends on the fire resistance class and the position of the reinforcing bars. Like the reduction factor \( k_a \), the reduction factor \( k_r \) has an upper and lower limit.

\[
A_m = 2 \cdot h + b_c = 2 \cdot 500 + 200 = 1200 \text{ mm}
\]
\[ V = h \cdot b_1 = 500 \cdot 200 = 100,000 \text{ mm}^2 \]

\[ u = \frac{1}{(1/u_1) + (1/u_u) + 1/(b_e - e_w - u_u)} \]

\[ = \frac{1}{(1/110) + (1/60) + 1/(200-10.2-60)} \]

\[ = 29.88 \text{ mm} \]

\[ k_e = \frac{(u \cdot a_3 + a_4) \cdot a_5}{\sqrt{A_v/V}} \approx \frac{(29.88 \cdot 0.026 - 0.154) \cdot 0.09}{\sqrt{1200/100,000}} = 0.51 \left( \begin{array}{c} > 0.1 \\ < 1.0 \end{array} \right) \]

**Table 6. Parameters for calculation of \( k_e \) (see EN 1994 Part 1-2, Annex F, Table F.5)**

<table>
<thead>
<tr>
<th>Standard fire resistance</th>
<th>( a_3 )</th>
<th>( a_4 )</th>
<th>( a_5 )</th>
<th>( k_{e,min} )</th>
<th>( k_{e,max} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>R 30</td>
<td>0.062</td>
<td>-0.16</td>
<td>0.126</td>
<td>0.1</td>
<td>1.0</td>
</tr>
<tr>
<td>R 60</td>
<td>0.034</td>
<td>-0.04</td>
<td>0.101</td>
<td>0.1</td>
<td>1.0</td>
</tr>
<tr>
<td>R 90</td>
<td>0.026</td>
<td>-0.154</td>
<td>0.090</td>
<td>0.1</td>
<td>1.0</td>
</tr>
<tr>
<td>R 120</td>
<td>0.026</td>
<td>-0.284</td>
<td>0.082</td>
<td>0.1</td>
<td>1.0</td>
</tr>
<tr>
<td>R 180</td>
<td>0.024</td>
<td>-0.562</td>
<td>0.076</td>
<td>0.1</td>
<td>1.0</td>
</tr>
</tbody>
</table>

To acquire the plastic moment resistance, the axial forces of the different parts should be determined.

Concrete:

\[ C_c = b_{eff} \cdot h_{e,h} \cdot \alpha_c \cdot f_c = 300.0 \cdot 10.9 \cdot 0.85 \cdot 2.5 = 6948.8 \text{ kN} \]

Upper flange:

\[ T_{f,u} = b_{h,u} \cdot e_f \cdot f_y = 12.4 \cdot 1.60 \cdot 35.5 = 704.3 \text{ kN} \]

Upper web:

\[ T_{w,u} = e_w \cdot h_t \cdot f_y = 1.02 \cdot 39.03 \cdot 35.5 = 1413.3 \text{ kN} \]

where:

\[ h_t = h - 2 \cdot e_f - h_1 = 50.0 - 2 \cdot 1.6 - 7.77 = 39.03 \text{ cm} \]

Lower web:

\[ T_{w,l} = e_w \cdot h_1 \cdot \left( 1 + k_{e,l} \right) \cdot f_y = 1.02 \cdot 7.77 \cdot \left( \frac{1 + 0.1}{2} \right) \cdot 35.5 = 154.7 \text{ kN} \]

\[ z_{w,l} = h_1 \cdot \frac{2 \cdot k_{e,l} + 1}{3 \cdot \left( k_{e,l} + 1 \right)} = 7.77 \cdot \frac{2 \cdot 0.1 + 1}{3 \cdot (0.1 + 1)} = 2.8 \text{ cm} \]

Lower flange:

\[ T_{f,l} = b \cdot e_f \cdot k_{e,l} \cdot f_y = 20.0 \cdot 1.6 \cdot 0.1 \cdot 35.5 = 113.6 \text{ kN} \]

Reinforcement bars:

\[ T_r = A_v \cdot k_{e,r} \cdot f_y = 14.1 \cdot 0.51 \cdot 50.0 = 359.6 \text{ kN} \]
Due to the fact that the compression force $C_c$ is larger than the sum of the tension forces $T$, the plastic neutral axis is situated in the concrete slab. So the plastic neutral axis is calculated to:

$$z_{pl} = \frac{\sum T_i}{\alpha_c \cdot f_c \cdot b_{eff}} = \frac{704.3 + 1413.3 + 154.7 + 113.6 + 359.6}{0.85 \cdot 2.5 \cdot 300} = 4.31 \text{ cm}$$

To get the moment resistance, the lever arms of the forces are needed:

Concrete slab (referring to upper edge of slab):

$$z_c = z_{pl}/2 = 4.31/2 = 2.16 \text{ cm}$$

Upper flange (referring to centre of gravity of concrete slab):

$$z_{f,u} = h_t + e_f/2 - z_c = 16.0 + 1.6/2 - 2.16 = 14.64 \text{ cm}$$

Upper web:

$$z_{w,u} = h_t + e_f + h_h/2 - z_c = 16.0 + 1.6 + 39.03/2 - 2.16 = 34.96 \text{ cm}$$

Lower web:

$$z_{w,l} = h_t + e_f + h_h + z_{w,l} - z_c = 16.0 + 1.6 + 39.03 + 2.8 - 2.16 = 57.27 \text{ cm}$$

Lower flange:

$$z_{f,l} = h_t + h - e_f/2 - z_c = 16.0 + 50.0 - 1.6/2 - 2.16 = 63.04 \text{ cm}$$

Reinforcement:

$$z_r = h_t + h - e_f - u - z_c = 16.0 + 50.0 - 1.6 - 11.0 - 2.16 = 51.24 \text{ cm}$$

The plastic moment resistance is determined to:

$$M_{pl,Rd} = T_{f,u} \cdot z_{f,u} + T_{w,u} \cdot z_{w,u} + T_{w,l} \cdot z_{w,l} + T_{f,l} \cdot z_{f,l} + T_r \cdot z_r$$

$$= 704.3 \cdot 14.64 + 1413.3 \cdot 34.96 + 154.7 \cdot 57.27 + 113.6 \cdot 63.04 + 359.6 \cdot 51.24$$

$$= 10,311 + 49,409 + 8860 + 7161 + 18,426$$

$$= 94,167 \text{ kNcm} = 942.7 \text{ kNm}$$

Verification:

$$\frac{810.0}{942.7} = 0.86 < 1 \quad \checkmark$$

REFERENCES


Example to EN 1994 Part 1-2: Composite column with partially encased steel sections

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1 TASK

The following example deals with a composite column made of partially encased steel sections. It is part of an office building and has a length of $L = 4.0$ m. In this example, the simple calculation model and the “tabulated data” method are used. The column is part of a braced frame and is connected bending resistant to the upper and lower column. Therefore the buckling length can be reduced as seen in Figure 1. The required standard fire resistance class for the column is R 60.

Figure 1. Buckling lengths of columns in braced frames

Figure 2. Cross-section of the column
Material properties:
Steel column:
- Profile: rolled section HE 300 B
- Steel grade: S 235
- Height: \( h = 300 \text{ mm} \)
- Width: \( b = 300 \text{ mm} \)
- Thickness of web: \( e_w = 11 \text{ mm} \)
- Thickness of flange: \( e_f = 19 \text{ mm} \)
- Cross-sectional area: \( A_a = 14900 \text{ mm}^2 \)
- Yield stress: \( f_y = 235 \text{ N/mm}^2 \)
- Elastic modulus: \( E_a = 210,000 \text{ N/mm}^2 \)
- Moment of inertia: \( I_z = 8560 \text{ cm}^4 \) (weak axis)

Reinforcement:
- Steel grade: S 500
- Diameter: 4 Ø 25
- Cross-sectional area: \( A_s = 1960 \text{ mm}^2 \)
- Yield stress: \( f_s = 500 \text{ N/mm}^2 \)
- Elastic modulus: \( E_s = 210,000 \text{ N/mm}^2 \)
- Moment of inertia: \( I_s = 4 \times 4.9 \times (30.0 / 2 - 5.0)^2 = 1960 \text{ cm}^4 \)
- Axis distance: \( u_s = 50 \text{ mm} \)

Concrete:
- Strength category: C 25/30
- Cross-sectional area: \( A_c = 300 \times 300 - 14900 - 1960 = 73,140 \text{ mm}^2 \)
- Compression strength: \( f_c = 25 \text{ N/mm}^2 \)
- Elastic modulus: \( E_{cm} = 30,500 \text{ N/mm}^2 \)
- Moment of inertia: \( I_c = 30 \times 30^3 / 12 - 8560 - 1960 = 56,980 \text{ cm}^4 \)

Loads:
- Permanent loads: \( G_k = 960 \text{ kN} \)
- Variable loads: \( P_k = 612.5 \text{ kN} \)

2 FIRE RESISTANCE OF A COMPOSITE COLUMN WITH PARTIALLY ENCASED STEEL SECTIONS

2.1 Mechanical actions during fire exposure
For fire design the accidental situation for combining loads is used:
\[
E_{\text{da}} = E \left( \sum G_k + A_d + \sum \psi_{2,i} \cdot Q_{\text{ki}} \right)
\]
With \( \psi_{2,1} = 0.3 \) the axial design load during fire exposure is:
\[
N_{fi,d} = 1.0 \cdot 960 + 0.3 \cdot 612.5 = 1143.8 \text{ kN}
\]

2.2 Verification using simple calculation model

2.2.1 Scope of application
The simple calculation model is a verification in the strength domain. It has to be verified, that the load at elevated temperatures is smaller than the design resistance.
\[
N_{fi,d} / N_{fi,Rd} \leq 1
\]
The design resistance for axial loads and buckling around the z-axis (weak axis) is calculated to:
\[ N_{f_i, R_d, z} = \chi_z \cdot N_{f_i, pl, R_d} \]

where:

- \( \chi_z \): Reduction factor depending on buckling curve \( c \) and non-dimensional slenderness
- \( N_{f_i, pl, R_d} \): Design value of the plastic resistance to axial compression in the fire situation

To use the simple calculation model, different constraints have to be fulfilled. Additionally, the column should be part of a braced frame.

### Table 1. Constraints for using simple calculation model

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Existing</th>
</tr>
</thead>
<tbody>
<tr>
<td>max ( l_\theta ) = 13.5 ( \cdot ) b + 0.3 = 4.05 m</td>
<td>( l_\theta = 0.5 \cdot 4.0 = 2.0 ) m ✓</td>
</tr>
<tr>
<td>230 mm ≤ h ≤ 1100 mm</td>
<td>( h = 300 ) mm ✓</td>
</tr>
<tr>
<td>230 mm ≤ b ≤ 500 mm</td>
<td>( b = 300 ) mm ✓</td>
</tr>
<tr>
<td>1% ≤ ( A_i / (A_i + A_j) ) ≤ 6%</td>
<td>19.6 / (731.4 + 19.6) = 0.03 = 3% ✓</td>
</tr>
<tr>
<td>max R 120</td>
<td>R 60 ✓</td>
</tr>
<tr>
<td>( l_\theta &lt; 10 \cdot b ) if ( 230 \leq b &lt; 300 ) or ( h/b &gt; 3 )</td>
<td>( b = 300 ) mm</td>
</tr>
<tr>
<td>( h/b = 300/300 = 1 )</td>
<td>( l_\theta = 0.5 \cdot 4.0 = 2.0 ) m ✓</td>
</tr>
</tbody>
</table>

#### 2.2.2 Calculation of the plastic design resistance and the effective flexural stiffness

According to Annex G of EN 1994 Part 1-2, the cross-section of the composite column is reduced. Some parts of the cross-section are reduced by reducing the cross-sectional area and some by reducing the yield stress and modulus of elasticity.

![Reduced cross-section for structural fire design](image)

The flanges of the steel profiles are reduced by determining reduction factors for the yield stress and the modulus of elasticity. For this, the average temperature of the flanges has to be calculated.

\[ \theta_{f,t} = \theta_{o,t} + k_i \cdot A_m / V \]
The temperature $\theta_{o,t}$ and the reduction factor $k_t$ are given in Table 2. The section factor is calculated as below:

$$\frac{A_m}{V} = \frac{2 \cdot (h + b)}{h \cdot b} = \frac{2 \cdot (0.3 + 0.3)}{0.3 \cdot 0.3} = 13.3 \text{ m}^{-1}$$

Table 2. Parameters for calculating the average flange temperature (see EN 1994 Part 1-2, Annex G, Table G.1)

<table>
<thead>
<tr>
<th>Standard fire resistance</th>
<th>$\theta_{o,t}$ [°C]</th>
<th>$k_t$ [m°C]</th>
</tr>
</thead>
<tbody>
<tr>
<td>R 30</td>
<td>550</td>
<td>9.65</td>
</tr>
<tr>
<td>R 60</td>
<td>680</td>
<td>9.55</td>
</tr>
<tr>
<td>R 90</td>
<td>805</td>
<td>6.15</td>
</tr>
<tr>
<td>R 120</td>
<td>900</td>
<td>4.65</td>
</tr>
</tbody>
</table>

For R 60, the average temperature arises to:

$$\theta_{f,t} = 680 + 9.55 \cdot 13.3 = 807 \text{ °C}$$

With this temperature, the reduction factors $k_{y,\theta}$ and $k_{E,\theta}$ are given in Table 3.2 of EN 1994 Part 1-2, where intermediate values can be interpolated linearly.

$$k_{y,\theta} = 0.06 + \left( \frac{(900 - 807) / (900 - 800)}{20} \right) \cdot (0.11 - 0.06) = 0.107$$

$$k_{E,\theta} = 0.0675 + \left( \frac{(900 - 807) / (900 - 800)}{20} \right) \cdot (0.09 - 0.0675) = 0.088$$

The plastic axial design resistance for the flanges and its flexural stiffness are determined to:

$$N_{f_i,pl,Rd,t} = 2 \left( b \cdot e_f \cdot k_{y,\theta} \cdot f_{ay,\theta} \right) / \gamma_{M_i,pl,a} = 2 \cdot (30 \cdot 1.9 \cdot 0.107 \cdot 23.5) / 1.0$$

$$= 286.65 \text{ kN}$$

$$\left( EI \right)_{fi,t,z} = k_{E,\theta} \cdot E_{a,f} \cdot \left( e_f \cdot b^3 \right) / 6 = 0.088 \cdot 21,000 \cdot (1.9 \cdot 30^3) / 6$$

$$= 1.58 \cdot 10^7 \text{ kNcm}^2$$

The web is reduced by its cross-sectional area and by the yield stress. The reduction of the height is calculated as below, where this height is reduced at both edges of the flange.

$$h_{w,f} = 0.5 \cdot \left( h - 2 \cdot e_f \right) \cdot \left( 1 - \sqrt{1 - 0.16 \cdot \left( H_t / h \right)} \right)$$

The parameter $H_t$ is given in Table 3.

Table 3. Parameter for reduction of the web (see EN 1994 Part 1-2, Annex G, Table G.2)

<table>
<thead>
<tr>
<th>Standard fire resistance</th>
<th>$H_t$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>R 30</td>
<td>350</td>
</tr>
<tr>
<td>R 60</td>
<td>770</td>
</tr>
<tr>
<td>R 90</td>
<td>1100</td>
</tr>
<tr>
<td>R 120</td>
<td>1250</td>
</tr>
</tbody>
</table>

Therefore, $h_{w,f}$ is calculated to:

$$h_{w,f} = 0.5 \cdot (30 - 2 \cdot 1.9) \cdot \left( 1 - \sqrt{1 - 0.16 \cdot (77/30)} \right) = 3.04 \text{ cm}$$

The yield stress is reduced to:

$$f_{ay,w,t} = f_{ay,w} \cdot \sqrt{1 - 0.16 \cdot (H_t / h)} = 23.5 \cdot \sqrt{1 - 0.16 \cdot (77/30)} = 18.04 \text{ kN/cm}^2$$
The axial design resistance and flexural stiffness for the web during fire exposure are:

\[
N_{fi,pl,Rd,w} = \left[ e_w \cdot \left( h - 2 \cdot e_f - 2 \cdot h_{w,fi} \right) \cdot f_{ay,w,t} \right] / \gamma_{M,fi,pl} \\
= \left[ 1.1 \cdot (30 - 2 \cdot 1.9 - 2 \cdot 3.04) \cdot 18.04 \right] / 1.0 \\
= 399.26 \text{kN}
\]

\[
(EI)_{fi,w,z} = \left[ E_{a,w} \cdot \left( h - 2 \cdot e_f - 2 \cdot h_{w,fi} \right) \cdot e_w^3 \right] / 12 \\
= \left[ 21,000 \cdot (30 - 2 \cdot 1.9 - 2 \cdot 3.04) \cdot 1.1^3 \right] / 12 \\
= 0.0047 \cdot 10^7 \text{kNcm}
\]

An exterior layer of concrete with a thickness \(b_{c,fi}\) is neglected in the calculation. This thickness is given in Table 4.

\[
\Rightarrow b_{c,fi} = 1.5 \text{ cm}
\]

### Table 4. Thickness reduction of concrete (see EN 1994 Part 1-2, Annex G, Table G.3)

<table>
<thead>
<tr>
<th>Standard fire resistance</th>
<th>(b_{c,fi} [\text{mm}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>R30</td>
<td>4.0</td>
</tr>
<tr>
<td>R 60</td>
<td>15.0</td>
</tr>
<tr>
<td>R 90</td>
<td>0.5 \cdot (A_m/V) + 22.5</td>
</tr>
<tr>
<td>R 120</td>
<td>2.0 \cdot (A_m/V) + 24.0</td>
</tr>
</tbody>
</table>

The rest of the concrete is reduced by the reduction factor \(k_{c,\theta}\) which depends on the temperature of the concrete. The average temperature of the concrete is given in Table 5. It depends on the section factor \(A_m/V\).

### Table 5. Average temperature of the concrete depending on the section factor (see EN 1994 Part 1-2, Annex G, Table G.4)

<table>
<thead>
<tr>
<th>(A_m/V [\text{m}^{-1}])</th>
<th>(\theta_{c,t} [\text{°C}])</th>
<th>(A_m/V [\text{m}^{-1}])</th>
<th>(\theta_{c,t} [\text{°C}])</th>
<th>(A_m/V [\text{m}^{-1}])</th>
<th>(\theta_{c,t} [\text{°C}])</th>
<th>(A_m/V [\text{m}^{-1}])</th>
<th>(\theta_{c,t} [\text{°C}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>136</td>
<td>4</td>
<td>214</td>
<td>4</td>
<td>256</td>
<td>4</td>
<td>265</td>
</tr>
<tr>
<td>23</td>
<td>300</td>
<td>9</td>
<td>300</td>
<td>6</td>
<td>300</td>
<td>5</td>
<td>300</td>
</tr>
<tr>
<td>46</td>
<td>400</td>
<td>21</td>
<td>400</td>
<td>13</td>
<td>400</td>
<td>9</td>
<td>400</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>50</td>
<td>600</td>
<td>33</td>
<td>600</td>
<td>23</td>
<td>600</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>54</td>
<td>800</td>
<td>38</td>
<td>800</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>41</td>
<td>900</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>43</td>
</tr>
</tbody>
</table>

\[
\Rightarrow \theta_{c,t} = 400 - \left( \left( 21 - 13.3 \right) / \left( 21 - 9 \right) \right) \cdot \left( 400 - 300 \right) = 336 \text{ °C}
\]

where:

\[
A_m/V = 13.3 \text{ m}^{-1},
\]

The reduction factor \(k_{c,\theta}\) and the strain \(\varepsilon_{cu,\theta}\) corresponding to \(f_{c,\theta}\) are given in Table 3.3 of EN 1994 Part 1-2.

\[
k_{c,\theta} = 0.75 + \left( \left( 400 - 336 \right) / \left( 400 - 300 \right) \right) \cdot \left( 0.85 - 0.75 \right) = 0.814
\]

\[
\varepsilon_{cu,\theta} = \left[ 10 - \left( \left( 400 - 336 \right) / \left( 400 - 300 \right) \right) \cdot \left( 10 - 7 \right) \right] \cdot 10^{-3} = 8.08 \cdot 10^{-3}
\]
The secant modulus of concrete can therefore be calculated to:

\[ E_{c,sec,\theta} = \frac{k_{e,\theta} \cdot f_c}{\gamma_{M,\beta,\sec}} = 0.814 \cdot 2.5 \left( \frac{8.08 \cdot 10^{-3}}{10} \right) = 251.9 \text{ kN/cm}^2 \]

The axial design resistance and the flexural stiffness of the concrete can now be determined:

\[ N_{fi,pl,rd,e} = 0.86 \cdot \left( \left( h - 2 \cdot e_f - 2 \cdot b_{c,\beta} \right) \cdot \left( b - e_w - 2 \cdot b_{c,\beta} \right) \right) - A \]

\[ \cdot f_c \gamma_{M,\beta,\sec} \]

\[ = 0.86 \cdot \left( \left( (30 - 2 \cdot 1.9 - 2 \cdot 1.5) \cdot (30 - 1.1 - 2 \cdot 1.5) \right) - 19.6 \right) \cdot (0.814 \cdot 2.5)/1.0 \]

\[ = 1017.3 \text{ kN} \]

\[ (EI)_{f,c,z} = E_{c,sec,\theta} \cdot \left( \left( h - 2 \cdot e_f - 2 \cdot b_{c,\beta} \right) \cdot \left( b - 2 \cdot b_{c,\beta} \right)^3 / 12 - I_{s,z} \right) \]

\[ = 251.9 \cdot \left( \left( (30 - 2 \cdot 1.9 - 2 \cdot 1.5) \cdot (30 - 2 \cdot 1.5)^3 - 1.1^3 \right) / 12 - 1960 \right) \]

\[ = 0.909 \cdot 10^7 \text{ kNcm}^2 \]

The reinforcing bars are only reduced by its yield stress and modulus of elasticity. The reduction factor \( k_{y,t} \) for the reduction of the yield stress is given in Table 6 and the reduction factor \( k_{E,t} \) for the reduction of the modulus of elasticity is gained from Table 7. Both are depending on the fire resistance class and the geometrical average \( u \) of the axis distances of the reinforcing bars to the outer borders of the concrete.

\[ u = \sqrt{u_1 \cdot u_2} = \sqrt{50 \cdot 50} = 50 \text{ mm} \]

where:

- \( u_1 \) the axis distance from the outer reinforcing bar to the inner flange edge
- \( u_2 \) the axis distance from the outer reinforcing bar to the concrete surface

Table 6. Reduction factor \( k_{y,t} \) for the yield stress \( f_{y,t} \) of the reinforcing bars (see EN 1994 Part 1-2, Annex G, Table G.5)

<table>
<thead>
<tr>
<th>Standard fire resistance</th>
<th>40</th>
<th>45</th>
<th>50</th>
<th>55</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>R 30</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>R 60</td>
<td>0.789</td>
<td>0.883</td>
<td>0.976</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>R 90</td>
<td>0.314</td>
<td>0.434</td>
<td>0.572</td>
<td>0.696</td>
<td>0.822</td>
</tr>
<tr>
<td>R 120</td>
<td>0.170</td>
<td>0.223</td>
<td>0.288</td>
<td>0.367</td>
<td>0.436</td>
</tr>
</tbody>
</table>

Table 7. Reduction factor \( k_{E,t} \) for the modulus of elasticity \( E_t \) of the reinforcing bars (see EN 1994 Part 1-2, Annex G, Table G.6)

<table>
<thead>
<tr>
<th>Standard fire resistance</th>
<th>40</th>
<th>45</th>
<th>50</th>
<th>55</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>R 30</td>
<td>0.830</td>
<td>0.865</td>
<td>0.888</td>
<td>0.914</td>
<td>0.935</td>
</tr>
<tr>
<td>R 60</td>
<td>0.604</td>
<td>0.647</td>
<td>0.689</td>
<td>0.729</td>
<td>0.763</td>
</tr>
<tr>
<td>R 90</td>
<td>0.193</td>
<td>0.283</td>
<td>0.406</td>
<td>0.522</td>
<td>0.619</td>
</tr>
<tr>
<td>R 120</td>
<td>0.110</td>
<td>0.128</td>
<td>0.173</td>
<td>0.233</td>
<td>0.285</td>
</tr>
</tbody>
</table>

\[ \Rightarrow k_{y,t} = 0.976 \]

\[ k_{E,t} = 0.689 \]
The plastic design resistance and the flexural stiffness of the reinforcing bars are calculated to:

\[ N_{f_i,pl,Rd,s} = A_y \cdot k_{y,s} \cdot f_{y,s} / \gamma_{M,f,i,s} = 19.6 \cdot 0.976 \cdot 50.0 / 1.0 = 956.5 \text{ kN} \]

\[ (EI)_{f_i,s,z} = k_{E,s} \cdot E_s \cdot I_{s,z} = 0.689 \cdot 21 \text{ 000} \cdot 1960 = 2.836 \cdot 10^7 \text{ kNcm}^2 \]

The design resistance all-over the cross-section is determined to:

\[ N_{f_i,pl,Rd} = N_{f_i,pl,Rd,f} + N_{f_i,pl,Rd,w} + N_{f_i,pl,Rd,c} + N_{f_i,pl,Rd,s} \]

\[ = 286.7 + 399.3 + 1017.3 + 956.5 \]

\[ = 2659.8 \text{ kN} \]

To calculate the effective flexural stiffness of the cross-section, reduction coefficients \( \phi_{i,\theta} \) have to be determined. They are given in Table 8.

### Table 8. Reduction coefficients for calculation of effective flexural stiffness (see EN 1994 Part 1-2, Annex G, Table G.7)

<table>
<thead>
<tr>
<th>Standard fire resistance</th>
<th>( \phi_{f,\theta} )</th>
<th>( \phi_{w,\theta} )</th>
<th>( \phi_{c,\theta} )</th>
<th>( \phi_{s,\theta} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>R 30</td>
<td>1.0</td>
<td>1.0</td>
<td>0.8</td>
<td>1.0</td>
</tr>
<tr>
<td>R 60</td>
<td>0.9</td>
<td>1.0</td>
<td>0.8</td>
<td>0.9</td>
</tr>
<tr>
<td>R 90</td>
<td>0.8</td>
<td>1.0</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>R 120</td>
<td>1.0</td>
<td>1.0</td>
<td>0.8</td>
<td>1.0</td>
</tr>
</tbody>
</table>

\[ (EI)_{f,eff,z} = \phi_{f,\theta} \cdot (EI)_{f_i,f,z} + \phi_{w,\theta} \cdot (EI)_{f_i,w,z} + \phi_{c,\theta} \cdot (EI)_{f_i,c,z} + \phi_{s,\theta} \cdot (EI)_{f_i,s,z} \]

\[ = 0.9 \cdot 1.58 \cdot 10^7 + 1.0 \cdot 0.0047 \cdot 10^7 + 0.8 \cdot 0.909 \cdot 10^7 + 0.9 \cdot 2.836 \cdot 10^7 \]

\[ = 4.70 \cdot 10^7 \text{ kNcm}^2 \]

#### 2.2.3 Calculation of the axial buckling load at elevated temperatures

The Euler buckling load or elastic critical load follows by:

\[ N_{f_i,cr,z} = \pi^2 \cdot (EI)_{f,eff,z} / l_\theta^2 = \pi^2 \cdot 4.70 \cdot 10^7 / (0.5 \cdot 400)^2 = 11610.7 \text{ kN} \]

where:

\( l_\theta \) buckling length of the column in the fire situation

The non-dimensional slenderness is obtained from:

\[ \overline{\lambda}_\theta = \sqrt{N_{f_i,pl,R} / N_{f_i,cr,z}} = \sqrt{2659.8 / 11610} = 0.48 \]

where:

\( N_{f_i,pl,R} \) the value \( N_{f_i,pl,Rd} \) with partial safety factors \( \gamma_{M,f,i} = 1.0 \)

The reduction factor \( \chi_z \) is determined by using buckling curve c of Table 6.1 of EN 1993 Part 1-1 and the non-dimensional slenderness in the fire situation.

\[ \chi_z = \frac{1}{\Phi + \sqrt{\Phi^2 - \overline{\lambda}_\theta^2}} = \frac{1}{0.68 + \sqrt{0.68^2 - 0.48^2}} = 0.86 \]

where:

\[ \Phi = 0.5 \cdot (1 + \alpha \cdot (\overline{\lambda}_\theta - 0.2) + \overline{\lambda}_\theta^2) = 0.5 \cdot (1 + 0.49 \cdot (0.48 - 0.2) + 0.48^2) \]

\[ = 0.68 \]
The buckling design resistance is calculated to:

\[ N_{\text{fi,Rd,z}} = \chi \cdot N_{\text{fi,pl,Rd}} = 0.86 \cdot 2659.8 = 2287.4 \text{ kN} \]

Verification:

\[ N_{\text{fi,d}} / N_{\text{fi,Rd,z}} = 1143.8 / 2287.4 = 0.50 < 1 \checkmark \]

2.3 Verification using tabulated data method

The verification using tabulated data is to be done in the strength domain.

When determining the load level \( \eta_{\text{fi}} \), the reinforcement ratio should be between 1% and 6%. Higher or lower ratios should not be taken into account.

\[
\begin{align*}
A_y & - 1\% \\
\frac{A_y}{A_y + A_s} & \leq 6\% \\
19.6 & \\
731.4 + 19.6 & = 0.03 = 3\% > 1\% \\
& < 6\% 
\end{align*}
\]

The load level is calculated to:

\[ \eta_{\text{fi,t}} = E_{\text{fi,d,t}} / R_d = N_{\text{fi,d}} / N_{\text{Rd}} = 1143.8 / 4130.4 = 0.28 \]

The parameters given in Table 4.6 of EN 1994-1-2 may be interpolated linearly. In this case it is not necessary to interpolate.

<table>
<thead>
<tr>
<th>Table 9. Verification of composite column with partially encased steel sections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
</tr>
<tr>
<td>---------------------------------------------------------------</td>
</tr>
<tr>
<td>( \min e_w / e_f = 0.5 )</td>
</tr>
<tr>
<td>( \min b = \min h = 200 \text{ mm} )</td>
</tr>
<tr>
<td>( \min u_s = 50 \text{ mm} )</td>
</tr>
<tr>
<td>( \min A_y / (A_y + A_s) = 4% )</td>
</tr>
</tbody>
</table>

The reinforcement ratio of the composite column is too low. To increase the ratio, reinforcement bars with bigger diameters or multiple reinforcement bars per corner can be applied.

However, the verification by using the simple calculation model could be accomplished successfully. This shows that the “tabulated data” method leads to conservative results.

REFERENCES